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Complex network effects on price setting: an agent-based computational approach♦

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Abstract

We present an adaptation of a new Keynesian model into an agent-based computational model, accounting for the importance of heterogeneity, interaction and bounded rationality in problems such as price setting and expectations formation. We evaluate the evolution of the distribution of price setting strategy frequencies, which agents might choose on each period between inflationary, neutral or deflationary, a process based on private and social features of agents' utility functions. In addition, we consider the network's topological structure and find that the regular network model fails in replicating exactly the new Keynesian model's original results, while the complex network model presents results in accordance with the originals.

Keywords

Agent-based model. Complex networks. Price setting.

Resumo

Apresentamos uma adaptação de um modelo novo keynesiano em um modelo computacional baseado em agentes, tendo em vista a importância da heterogeneidade, interação e racionalidade limitada nos problemas como fixação de preços e formação de expectativas. Avaliamos a evolução da distribuição de frequência das estratégias de formação de preços, que os agentes

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podem escolher a cada período entre inflacionária, neutra ou deflacionária, processo este baseado nas características privadas e sociais da função utilidade dos agentes. Adicionalmente, levamos em conta a estrutura topológica da rede em conta, e verificamos que o modelo em rede regular fracassa em replicar com exatidão os resultados originais do modelo novo keynesiano, enquanto o modelo em rede complexa apresenta resultados congruentes com os originais.

Palavras-chave
Modelos baseado em agentes. Redes complexas. Formação de preços.

Classificação JEL
C63, D80, E30.

1. Introduction

Price setting and expectations are subject to long-standing debates in macroeconomics. Most new Keynesian models assume that individuals formulate expectations independently and employ a representative agent to analyse them, assuming that individuals have similar preferences, access to the same information and the same reasoning prowess. It goes without saying that these assumptions are not to be taken literally, but they are the foundations on which canonical new Keynesian models are built.

However, these models have its critics. Flieth and Foster (2002) for instance remark that representative agent models face hardships trying to reproduce patterns found empirically. With that in mind, we have agent-based models (ABM), a newer style of modelling that, according to Macal and North (2005), can be an important tool to evaluate social systems composed by different agents that interact amongst them, learn with past experiences, influence each other and adapt their behaviour looking for the best responses to their environment.

Recent literature such as Fainmesser and Galeotti (2016) or Fainmesser and Galeotti (2020) highlight that network effects can be very important in price setting, with firms and consumers looking at their neighbourhoods to gather information in order to make better decisions. The amount of influence these relations exert depend on various factors, like the strength of network effects, network size and the behaviour of a few very influential vertices.

This study's proposed model is an ABM which takes network effects into account, assuming a monopolistic competition economy based on Ball and Romer (1989). In our reasoning, agents must choose their strategy on each round: keeping the same prices, raising them or lowering them. Their choices depend on their preferences. These agents are disposed in a network, which starts out regular (where neighbourhoods are always the same) but eventually will include a degree of randomness, as proposed by Watts and Strogatz (1998).

Therefore, this article's main contribution to macroeconomic research is assessing whether the conclusions put forward by Ball and Romer (1989) still hold up after relaxing certain hypotheses – namely, representative agents, perfect rationality, and a tendency towards symmetrical equilibria. These hypotheses are put aside in favour of heterogeneous agents, bounded rationality and a non-enforcement of equilibria.

Thus, this article is structured as follows: next section is a review of the literature touching on topics such as price setting, social interaction and networks in economics; the third section lays out the methodology behind the model here proposed; the fourth section presents and discusses the results achieved; and the fifth and final section presents our concluding remarks.

2. Prices, social interaction and networks

2.1. Some theory and evidence on price setting

This subsection assembles some findings regarding price setting and expectations that are of great interest to us due to how they relate to the model we will run afterwards. As a starter, the study published by Levy et al. (1998) about the price adjustment process at multiproduct retail stores shows that this is a more complex task than previously thought. Therefore, at some moments it isn't rigorously planned, resulting in price stickiness. The authors remark that this is caused mainly by item pricing laws and the fear that agents have of making mistakes while setting prices.

According to Nakamura (2008), 16% of the variation in product prices is common all across the market, while 65% is common only within a same retail chain and 17% of it is completely idiosyncratic. Midrigan (2011) too finds that it's more likely that a particular product has its price changed if a big fraction of the other prices within the same store are being changed, especially when the price changes touch similar goods. He also points out (although based on weaker evidence than on the previous finding) that there is some level of synchronization across stores in a given city.

In this article we chose an evolutionary approach when discussing price setting. Bonomo et al. (2003), for instance, employ evolutionary games to evaluate the transaction costs between two equilibria with the goal of reaching disinflation. Based on their findings we can assume that agents with traits of bounded rationality suffer worse losses than more rational agents and strategies with below average performance have their use reduced with the passage of time. Likewise, in the long run they will eventually be able to learn which strategy leads to more gains. In the same vein, Saint-Paul (2005) aims to find a motivation for price stickiness. The author aims to answer two main questions: (i) if the economy converges to a rational expectations equilibrium; (ii) in the case this doesn't happen, if is it possible to find a particular behaviour of sticky prices. The author builds a model where economic agents are imperfectly rational and interact in a localized manner with others. The author argues that, even though the rational expectations equilibrium is among the possible choices by agents, the economy doesn't necessarily converge to this equilibrium.

Still on evolutionary dynamics, Silveira and Lima (2008) elaborate a model that takes into account the emergence of monomorphic (where only one price-setting strategy survives between perfect and bounded rationality) and polymorphic (both survive in the long run) equilibria. Another difference is that while on the monomorphic equilibria the money is neutral, on polymorphic equilibria the money might not be.

Another model from Lima and Silveira (2015) allows firms to choose the Nash strategy (update their information and establish optimal prices) by paying a cost, or else they can choose the bounded rationality strategy (which is free) with lagged information. Evolutionary dynamics take the process to a long-run equilibrium where, even though most or all firms employ the bounded rationality strategy, the price level is the same as the one from the symmetric Nash equilibrium.

2.2. Social interaction in economics

In an effort to bring theory closer to empirical data, research in economics has been more observing of the importance of social interaction between economic agents. Hommes (2006) reminds that, in a social interaction framework, the payoff received by a certain agent for his/her actions is directly related to the agents next (be it in a geographical, economic or social sense) to him/her. Their gains do not come only through market mechanisms, but also through imitation, learning and peer pressure.

Flieth and Foster (2002) notes that the expectation formation process involves discussion amongst agents, as an individual has his/her expectations influenced by opinions of friends, business partners or competitors. Topa (2001) highlights those interactions happen between social neighbours which share certain socioeconomic proximity, but not necessarily geographic: this could mean individuals who took the same classes at school, co-workers or any attendants of the same social activities, even if they do not live close to each other.

If a firm decides to raise the price of a good it produces, it cannot raise the price much higher than the neighbouring firms that produce close substitute goods, as that raises the chance of losing clients, therefore not maximising profits. Bernhardt (1993) points out there is a cost in price adjustment, in the sense that not coordinating with close firms means that an increase in price might in fact not be profit maximising: it would be only a question of following a nominal change in money supply.

In this context, Hohnisch et al. (2005) propose an interactive expectations model to study business environment amongst entrepreneurs. Each entrepreneur is faced with a ternary choice between negative, neutral or positive expectations. Thus, it would change its evaluation of business climate based on the opinions of its neighbours, with the results emulating with great proximity some traits of the German business confidence index.

Fainmesser and Galeotti (2016) develop a model with a monopolistic firm that sells a network good and discriminate prices by using information about consumers' influence and their susceptibility to peer pressure. The monopoly's maximising behaviour happens when offering discounts to influential consumers and charging premia to susceptible clients (that are more likely to go along side influential consumers). Fainmesser and

Galeotti (2020) make new progress by showing that this influence marketing leads to inefficient consumer-product matches, making firms invest in more information. This competition for information brings firms' profits closer to zero but, at the same time, increases consumer surplus.

In macroeconomics, with the prevalence of the new neoclassical synthesis and DSGE models, starting studies in this field were rather timid but, more recently, models with social interaction have been getting more attention. The reason behind it is that current mainstream models have started to come under fire since the subprime crisis, facing difficulties when trying to fit empirical findings in their framework, as Romer (2016) staunchly points out. ABMs, however, present promising tools to address these issues.

Dosi and Roventini (2019) point that The Great Recession was a natural experiment in macroeconomics, considering it showed issues with the mainstream approach. It displayed that price stickiness, for instance, can be better approached under the lens of agent-based modelling. The author signals that price stickiness might arise from heterogeneity, imperfect information and coordination hurdles.

However, not all is lost for mainstream economics, as there is a recent strife between RANK (Representative Agent New Keynesian) and HANK (Heterogeneous Agent New Keynesian) models. The mainstream models pointed out in the last paragraphs as suffering from lack of realism are the RANK models. In similar contexts, HANK models performed better, as discussed by Kaplan et al. (2018) and Acharya et al. (2020).

For instance, Kaplan et al. (2018) hint that, in the face of a Ricardian exchange failure, HANK models capture better the monetary policy transmission mechanism to households. In the same vein, Acharya et al. (2020) stress out that, in HANK models, optimal monetary policy is considerably different from the standard (derived from RANK models), due to household inequalities. However, even though heterogeneity and sometimes bounded rationality is accounted for in these models, network effects are still mostly ignored.

2.3. Elements of network theory

This subsection is a primer on network science, as some basic concepts need to be laid out to better understand the computational model used here. Goyal (2012) explains that the networks of a given system might be represented mathematically through graph theory, according to which a network might be represented by $G = (N, B)$ in which N is a set (non--empty and finite) of nodes and B is the set of edges, formed by unordered pairs of distinct nodes.

It can be said that two nodes i and j are directly connected if they are adjacent to each other. If they are not adjacent, but are part of a sequence of adjacent nodes linked amongst themselves, then i and j are indirectly connected. The neighbourhood of any given node is composed by a set formed by nodes connected by edges to this node. In complex networks, a node can connect with another one very far from its geographical location.

This means nodes can be linked even when their positions in the network are far from each other, and nodes that are very far from each other might be linked in a sequence with a few steps (small-world phenomena). In order to represent complex networks, Watts and Strogatz (1998) suggest the usage of ring networks (see Figure 1), where nodes are in a circumference and uniformly distributed throughout it.

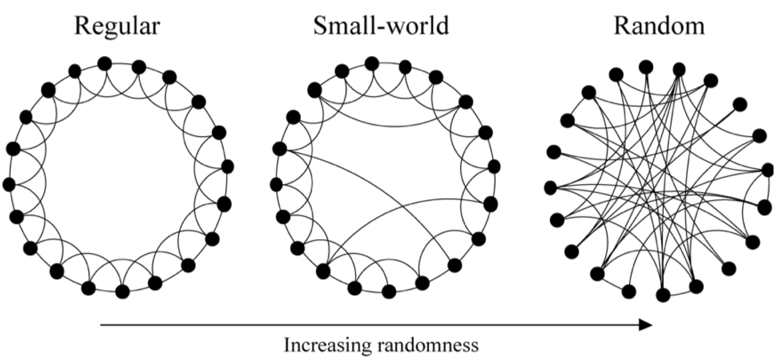

Figure 1 – Complex network types
Source: Watts and Strogatz (1998).

Each of the nodes is linked to K neighbours in clockwise sense and K neighbours in anticlockwise sense. Having defined a ring network, we can eliminate an edge between the i-th node and a neighbour, and then create a new link between i and any other node chosen randomly with a probability $p \in [0, 1] \subset \mathbb{R}$. This operation is called *rewiring*.

A network is called regular when $p = 0$, and its main feature is having the same number of edges for all nodes. For any i node, in a regular network the connected edges are always the same. If $p = 1$ the network is called random. Watts and Strogatz (1998) remark that, empirically, social networks tend to present high clustering but a low average distance between nodes. As it has been pointed out by the authors, these traits put many empirical networks halfway between the regular and random networks, and in case $p \approx 0, 1$ we can call it a small-world network. It has an average distance between nodes comparable to the average distance of a random network, but the clustering is strictly superior to the one from a random network.

3. A price-setting game on a complex network

3.1. A discrete choice model

We will now build the model, starting with an explanation on how agents make decisions. Consider an agent i that must choose between three mutually exclusive alternatives, denoted by -1, 0 and 1. Let $\sigma_i \in \{-1, 0, 1\}$ be the i-th agent's choice in period $t \in N$. At any period t, each agent can choose between lowering one's price ($\sigma_i = -1$), maintaining the same price ($\sigma_i = 0$) or raising it ($\sigma_i = 1$).

Train (2009) remarks that preferences (and, consequently, the utility function that represents them) depend on observable motivations and non-observable motivations, the last depending on idiosyncratic traits of an agent. Due to non-observable motivations, decision-making phenomena is stochastic, not deterministic, to someone watching from the outside. To account for that, the utility function will feature a deterministic component referring to observable traitsand a stochastic one that is associated to non-observable traits:

$$\mathcal{U}(\sigma_i) = \mathcal{U}^d(\sigma_i) + \varepsilon(\sigma_i), \tag{1}$$

where $\mathcal{U}^d(\sigma_i)$ is the function's deterministic component, associated to observable motivations, and $\varepsilon(\sigma_i)$ the stochastic component, associated to non-observable motivations. After defining the utility function, next step is evaluating the agent's optimal choice. Choosing a certain value $\sigma_i \in \{-1, 0, 1\}$ is the agent's optimal choice in case it fulfills the following condition:

$$\mathcal{U}(\sigma_i) \geq \mathcal{U}(\sigma_i'), \forall \sigma_i' \in \{-1,0,1\} \tag{2}$$

It is possible to rewrite (2) using (1) as follows:

$$\mathcal{U}^d(\sigma_i) - \mathcal{U}^d(\sigma_i') \geq \varepsilon(\sigma_i') - \varepsilon(\sigma_i), \forall \sigma_i' \in \{-1,0,1\}, \tag{3}$$

in a way to highlight that choosing a certain σ_i will be an optimal choice for the *i*-th agent if the net gain from the observable component (left-hand side) is bigger than the net gain from the stochastic component (right-hand side), associated to any σ_i' alternative.

However, even if the observable utility of a given strategy σ_i is bigger than the others, that does not guarantee that σ_i will be chosen by the *i*-th agent. Non-observable incentives from one of the other strategies might be bigger than the ones from the σ_i strategy. Thus, it is only possible to define the probability of the *i*-th agent choosing $\sigma_i \in \{-1, 0, 1\}$ from (2) and (3):

$$\begin{aligned} Prob(\sigma_i) &= Prob(\mathcal{U}(\sigma_i) \geq \mathcal{U}(\sigma_i')\ \forall \sigma_i'), \\ &= \int_{-\infty}^{\infty} I\left[\varepsilon(\sigma_i') - \varepsilon(\sigma_i) \leq \mathcal{U}^d(\sigma_i) - \mathcal{U}^d(\sigma_i')\ \forall \sigma_i'\right] f(\vec{\varepsilon_i}) d\vec{\varepsilon_i}, \end{aligned} \tag{4}$$

where $f(\vec{\varepsilon_i})$ is the joint probability density function (PDF) of the random variables vector $\vec{\varepsilon_i} = \varepsilon(\sigma_i = -1), \varepsilon(\sigma_i = 0), \varepsilon(\sigma_i = 1)$ and $I[\cdot]$ an indicator function, equal to 1 if the inequality between brackets is true and zero if false. What follows is that function (4) is a cumulative density function (CDF) of the utility function's random component given by equation (1). This CDF indicates the *i*-th agent's propensity towards choosing a certain strategy $\sigma_i \in \{-1, 0, 1\}$. The propensity towards choosing σ_i raises together with the difference between observable incentives, meaning idiosyncratic motivations have their importance diminished when this differential raises.

Train (2009) highlights that diverse discrete choice models might be derived from distinct specifications of the PDF given by $f(\vec{\varepsilon}_i)$, with the most common being the one whose final result is the logit model. To do so, suppose the random components of (1) are random variables independent amongst themselves, with the same probability distribution for extreme values. This means that for each $\varepsilon(\sigma_i)$, the PDF is a Type-1 Gumbel formally given by:

$$f\big(\varepsilon(\sigma_i)\big) = \beta e^{\left(-\beta\varepsilon(\sigma_i)\right)} e^{\left(-e^{-\beta\varepsilon(\sigma_i)}\right)}, \tag{5}$$

with $\beta > 0$ being a real constant. The CDF corresponding to the PDF given by (5) is in turn given by:

$$F\big(\varepsilon(\sigma_i)\big) = e^{\left(-e^{-\beta\varepsilon(\sigma_i)}\right)}, \tag{6}$$

and by inserting (5) and (6) in (4) it is possible to obtain the logistic CDF, defined by the following equation:

$$\begin{aligned} Prob(\sigma_i) &= \frac{e^{\beta U^d(\sigma_i)}}{e^{\beta U^d(\sigma_i)} + e^{\beta U^d\left(\sigma_i'\right)} + e^{\beta U^d\left(\sigma_i''\right)}}, \\ &= \frac{1}{1 + e^{-\beta\left[U^d(\sigma_i) - U^d\left(\sigma_i'\right)\right]} + e^{-\beta\left[U^d(\sigma_i) - U^d\left(\sigma_i''\right)\right]}}, \end{aligned} \tag{7}$$

where σ_i, $\sigma i'$ e $\sigma i''$ are three distinct choices.

An interpretation of β relevant to analysing the results is offered by Brock and Hommes (1997), who remark that, *ceteris paribus*, when β is small, the impact of non-observable incentives is larger on the probability of choosing a certain strategy. Therefore, choices little depend on deterministic utility. However, if β has a bigger value, the strategy chosen will probably be the one that offers the biggest deterministic utility.

Brock and Durlauf (2001) propose including a third term on the function (1) to represent network effects. In this logic, the i-th agent makes his/her decisions influenced by his/her social neighbourhood, henceforth called n_i. Formally, n_i is the set of agents whose decisions are observed by the *i*-th agent and influence his/her decisions. Including network effects as an observable incentive causes the deterministic component of the utility function to be represented as follows

$$U^d(\sigma_i) = \alpha\,\mathcal{U}^{\wp}(\sigma_i) + \mathcal{U}^{s}\left(\sigma_i, \overrightarrow{\sigma_i^e}\right), \tag{8}$$

with α being a parametric constant that measures the relative weight of deterministic private utility, $\mathcal{U}^{\wp}(\cdot)$, representing all observable incentives except network effects, and $\mathcal{U}^{s}(\cdot)$ being the social deterministic utility that suffers network effects. It is worth highlighting that $\mathcal{U}^{s}(\cdot)$ depends not only on the choice from the *i*-th agent but also on the choices from the n_i neighbourhood, represented by the vector $\sigma^e \equiv (\sigma^e)_{j\in n}$.

Inserting (8) in (1), we can obtain a new equation for the utility of the *i*-th agent when he/she chooses σ_i:

$$\mathcal{U}\,(\sigma_i) = \alpha\mathcal{U}^{\wp}(\sigma_i) + J\,U^s(\overrightarrow{\sigma_i}) + \varepsilon(\sigma_i) \tag{9}$$

and after this adaptation, employing the same reasoning used when deriving equation (7), we can obtain the probability of choosing the σ_i alternative but with $U^d(\sigma_i)$ now defined by (8).

Lastly, the social deterministic utility $\mathcal{U}^{s}(\cdot)$ will be derived. It depends on the network topology, but for demonstration ends, a regular network with quadratic form (square lattice) – due to ease of computational implementation – and *N* agents will be assumed. Therefore, the n_i social neighbourhood of the *i*-th agent located in the coordinates $(\ell, c) \in \{1, \dots, N\}X\{1, \dots, N\}$ of order *N* will be defined by:

$$n_i = \{(m, n) \in \{1,2, \dots, N\}^2 \colon |k - m| + |l - n| = 1\} \tag{10}$$

After defining n_i, it is possible to define the social utility of the *i*-th agent located in the coordinates $(\ell, c) \in \{1, \dots, N\}X\{1, \dots, N\}$ as:

$$\mathcal{U}_i^{s}(\overrightarrow{\sigma_i}) = \tfrac{J}{4}\textstyle\sum_{j\in n_i} \delta_{\sigma_i\sigma_j}, \tag{11}$$

where $J > 0$ is a parametric constant that measures the degree of influence in the neighbourhood and $\delta_{\sigma_i\sigma_j}$ is the Kronecker delta. If $\sigma_i = \sigma_j$, then $\delta_{\sigma_i\sigma_j} = 1$. Otherwise, $\delta_{\sigma_i\sigma_j} = 0$, so that the Kronecker delta can be rewritten as:

$$\delta_{\sigma_i\sigma_j} = \sum_{j\in n_i}\frac{1}{2}\left[\sigma_i\sigma_j + 3\sigma_i^2\sigma_j^2 - 2\left(\sigma_i^2 + \sigma_j^2\right) + 2\right] \tag{12}$$

Substituting (12) in (11), it is possible to redefine social utility as:

$$\mathcal{U}_i^s(\sigma_i) = \frac{J}{8}\sum_{j\in n_i}\left[\sigma_i\sigma_j + 3\sigma_i^2\sigma_j^2 - 2\left(\sigma_i^2 + \sigma_j^2\right) + 2\right] \tag{13}$$

In short, if $\alpha/J > 1$ then agents will put more weight on private utility, else they will be more affected by network incentives. As for β, the lower it is, the more random will the choice be. This includes bounded rationality in the model, as an agent might make bad choices from a deterministic utility viewpoint, choosing a worse strategy if he/she puts a low weight on observable incentives.

3.2. Adaptation of a new Keynesian price-setting model

This subsection briefly introduces the model conceived by Ball and Romer (1989) and adapts it into a network model. Let us have N producers that fabricate distinct goods that are produced by their own labour (thus suppressing labour markets). They are subsequentially sold and profits are used to purchase products from other producer-consumers – the goods, however, are close substitutes amongst themselves. With this model as basis, we can swap rational expectations for interactive expectations by sorting heterogeneous agents in the network.

In the original model, money is just a medium of exchange to make transactions, so it is possible to employ money supply, M, as a proxy of aggregate nominal demand. Therefore, the producer's maximum utility as a function of the nominal aggregate demand, the price of good i, P_i, and the general price level, P, resulting from the simultaneous decisions of all producers can be written as follows:

$$U_i = \frac{M}{P}\left(\frac{P_i}{P}\right)^{1-\epsilon} - \left(\frac{\epsilon-1}{\gamma\epsilon}\right)\left(\frac{M}{P}\right)^{\gamma}\left(\frac{P_i}{P}\right)^{-\gamma\epsilon} \tag{14}$$

Therefore, the i-th producer's optimal price, P_i^*, is that which maximises his/her utility is given by:

$$P_i^* = P^{\phi} M^{1-\phi}, \tag{15}$$

where $\phi = \frac{1+(1-\epsilon)(1-\gamma)}{1-\epsilon(1-\gamma)} \in (0,1) \subset \mathbb{R}$ represents the elasticity of the i-th producer's optimal price with respect to the general price level.

In the price-setting game described by Ball and Romer (1989), a symmetric Nash equilibrium occurs when $P_i^* = P$ for all $i = 1, 2, \ldots, N$. If that happens, then $P_i^* = P = M$, $C_i = C = 1$ and $Y_i = 1$ for all $i = 1, 2, \ldots, N$.

Relaxing the hypothesis that states that producers immediately identify the game's symmetric Nash equilibrium, it is possible to adapt the model here described to a network game. This means that in period t, the i-th producer, when choosing the price $P_{i,t}$, must form an expectation of the general price level P_t to apply the optimal rule, which is given by:

$$P_{i,t} = \left(P_{i,t}^{e}\right)^{\phi} (M_t)^{1-\phi}, \tag{16}$$

where $P_{i,t}^{e}$ is the general price level expected on period t by the i-th producer.

As mentioned beforehand, one of the main changes is abandoning the rational expectations paradigm, where $P_{i,t}^{e} = P_t$, employing bounded rationality instead, meaning that agents might not always make the best decision. They need to choose their stance on each step, either lowering the price ($\sigma_{i,t} = -1$), maintaining it ($\sigma_{i,t} = 0$) or raising it ($\sigma_{i,t} = 1$). To make a choice, an agent will choose the strategy which grants the most overall utility.

Now we will present the utility function's three components. Drawing on Silva (2012), deterministic private utility is given by:

$$\mathcal{U}^{\wp}(\sigma_{i,t}) = \left(\frac{P_t - P_{t-1}}{P_{t-1}}\right) \sigma_{i,t} \tag{17}$$

Notice that for $U^{p}(\sigma_{i,t}) > 0$, both terms multiplying amongst themselves must have the same signal, meaning that if the agent reduces his/her price ($\sigma_{i,t} = -1$) and the general price level falls ($P_t < P_{t-1}$), it follows that $U^{p}(\sigma_{i,t}) > 0$. Analogously, the same happens in case the agent raises his/her price and the general price level goes up. If the agent is neutral ($\sigma_{i,t} = 0$), deterministic private utility follows the price variation's signal.

As for the deterministic social utility, for the i-th producer on period t, it depends on the strategies of the producer himself and also of the n_i neighbourhood. Deterministic social utility is given by:

$$\mathcal{U}^s(\sigma_{i,t}) = \frac{J}{8}\sum_{j\in n_i}\left[\sigma_{i,t}\sigma_{j,t} + 3\sigma_{i,t}^2\sigma_{j,t}^2 - 2\left(\sigma_{i,t}^2 + \sigma_{j,t}^2\right) + 2\right] \tag{18}$$

As outlined before, the stochastic component of the utility function will be given by random variables independent amongst themselves that follow the Type-1 Gumbel distribution, formally defined in equation (5). Based on these assumptions, it is possible to express the propensity to choose a certain strategy $\sigma_{i,t}$ in the following way:

$$Prob\left(\sigma_{i,t+1}\right) = \frac{1}{1 + e^{-\beta\left[U^d(\sigma_{i,t}) - U^d(\sigma'_{i,t})\right]} + e^{-\beta\left[U^d(\sigma_{i,t}) - U^d(\sigma''_{i,t})\right]}}\ . \tag{19}$$

The last step for model closure is defining how producers, given their states on period t, form their expectations for the general price level, $P_{i,t}^e$. If the agent is neutral, $P_{i,t}^e = P_{t-1}$, if the agent is inflationary, $P_{i,t}^e$ will be chosen randomly between the real interval $[P_{t-1}, 1.2\,P_{t-1}]$, and if the agent is deflationary, $P_{i,t}^e$ will be chosen randomly between the real interval $[0.8\,P_{t-1}, P_{t-1}]$.

In short, the above options can be described intuitively. Drawing again on Silva (2012), let us assume $P_{i,t}^e$ be a uniformly distributed random variable, with its valued conditioned to the choice of $\sigma_{i,t}$. More precisely, the value of each agent's expected $P_{i,t}^e$ will be chosen amongst:

$$\begin{cases} \widetilde{P_{i,t}} \in [0.8P_{t-1}, P_{t-1}], \text{ if } \sigma_{i,t} = -1, \\ P_{t-1}, \text{ if } \sigma_{i\,t} = 0, \\ \widetilde{P_{i,t}} \in [P_{t-1}, 1.2P_{t-1}], \text{ if } \sigma_{i,t} = 1. \end{cases} \tag{20}$$

Based on their expectations of the general price level, each agent will define an individual price on t by using the optimal rule given by equation (16), whereas the general price level is calculated by the following equation:

$$P = \left(\frac{1}{N}\sum_{j=1}^{N} P_j^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{21}$$

An outcome that could not happen in the original model by Ball and Romer (1989) can and will happen all the time in ours: at any given moment, there can be heterogeneity amongst producer prices and a general price level that is not the symmetric Nash equilibrium price level which would be reached assuming rational expectations.

Some variables of interest for the results will be the output level and price variance. The authors define the money supply $M = PC$, and as the model does not mention investment or government expenditure we can, for each period, isolate the mean consumption index and treat it as the aggregate output:

$$Y_t = \frac{M_t}{P_t} \tag{22}$$

and lastly, the price variance on each step can be described by the standard formula:

$$Var_t = \frac{1}{N}\sum_{i=1}^{N}\left[P_{i,t} - P_t\right]^2 \tag{23}$$

3.3. Computational implementation

This subsection provides details on the initial conditions setup, the computational loop and how agents are disposed in the program. To implement different network structures, we can do a rewiring procedure as described by Watts and Strogatz (1998), but using a matrix instead of a ring network due to ease of programming. This is not a problem, as we can program agents in the first row and column to be linked respectively to the last row and column to solve the contour issue. We will assume N = 10, 000 agents for our simulations, which is a 100X100 matrix.

For this procedure to be formally implemented to a matrix structure, we can employ the shortcuts routine as described by Taylor and Higham (2009). That is, there will be a rewiring probability $p \in [0, 1] \subset \mathbb{R}$, in which a shortcut will be generated between two nodes that are not immediate neighbours – the larger p is, the more random a network will be. A shortcut means that an agent's link towards one of his/her neighbours is severed and the agent connects with another peer from anywhere in the matrix.

Agents will only change strategies on the start of each step. As the strategies distribution has a certain weight in producer decisions, the initial conditions are that in $t = 0$ each one of the strategies starts with 33.33% of adherence by the agents, distributed randomly. Also, using a uniform probability distribution, prices are spread randomly inside the interval . $[0.8P_{t-1}, 1.2P_{t-1}] \subset \mathbb{R}$

We have chosen 20% price increases at most due to evidences shown by authors such as He et al. (2010) and Roberts and Supina (1996) that most goods prices go up or down by 20% in the short run, no more than that. After defining individual prices, the general price level is set for the $t = 0$ period, using equation (21) and monetary supply is fixed as 1.

Having established the initial conditions, period t = 1 starts. With the propensities toward strategies already defined, producers form their expected prices ($P^e_{i,t}$), as shown by equation (20). After expected prices are established, each agent's individual price is obtained based on equation (15). When a period ends, firms are made aware of general price level, output level and variance between individual prices compared to the general price level. With this information and observing strategies in their vicinity, we are able to calculate each agent's utility function, according to equation (8).

After calculating utilities, the logistic CDF in (7) is taken to measure the propensity of choosing each of the three strategies. Afterwards, a number $r_{i,t}$ contained inside the interval $[0, 1] \subset \mathbb{R}$ will be randomly generated. Let us have $\sigma_{i,t} = 0$, $\sigma i',t = 1$ and $\sigma i'',t = -1$. If $r(i, t) \leqslant \text{Prob}(\sigma_{i,t} = 0)$ the producer will adopt a neutral strategy, else if $\text{Prob}(\sigma_{i,t}=0) < r \leqslant \text{Prob}(\sigma_{i,t}= 0) + \text{Prob}(\sigma_{i,t} =1)$ the producer will adopt an inflationary strategy. Finally, If $r > \text{Prob}(\sigma_{i,t} = 0)+\text{Prob}(\sigma_{i,t} = 1)$, the producer will adopt a deflationary strategy. Afterwards, a new period starts.

3.4. Model calibration

This subsection provides an insight into the model's calibration process. We will calibrate the model aiming to find parameter sets that create a synthetic time series similar to the empirical time series. As the original model is populated by producers, we chose the Producer Price Index from the United States, in particular the finished goods consumption index, as the model deals with their pricing. Monthly data was obtained from the Federal Reserve of St. Louis for the period from 04/1947 (first entry in the series) until 12/2020, totalling 884 observations.

The calibration criterion used was minimising the sum of squared errors, which consists of finding the set of parameters that minimises the squared difference between empirical and simulated prices, formally given by:

$$\frac{1}{T}\sum_{t=1}^{T}\left(P_{e,t} - P_{s,t}\right)^2 \tag{24}$$

with T being the number of periods, P_e the price index from empirical data and P_s the general price index from simulated data.

Based on this criterion, an optimisation algorithm will be employed to find the best-fitting combination of parameters $\{\phi, \alpha, \beta, J, p\}$ that minimises equation (24), using a variety of Newton's methods in order to find the answers to the optimisation problem. Given the initial guesses plus the inferior and superior bounds of the parameters to be calibrated, a random set of parameters is chosen, generates simulated values and compares them to real ones. If the set generates a smaller distance than before, the function stores the new parameters and discards the previous, repeating the process until finding parameters that minimise the error.

4. Results and emergent properties of the model

4.1. The benchmark case: a price-setting game on a regular network

In this section we will present the results. For the sake of comparison, we will start with the most basic case, which is a regular network model – thus, $p = 0$. We calibrate the model using the procedure described in Subsection 3.4. To avoid using extreme results from only one run, all results are averaged from 30 simulations, except for the parameter tests from section 4.3.

Table 1 – Calibrated values for the regular network model

Parameter	Calibrated value	Range of possible values
α	0.01245	$[0, 5] \subset \mathbb{R}$
β	2.0954	$[0, 5] \subset \mathbb{R}$
ϕ	0.7653	$[0, 1] \subset \mathbb{R}$
J	6.9351	$[0, 10] \subset \mathbb{R}$

Source: own elaboration.

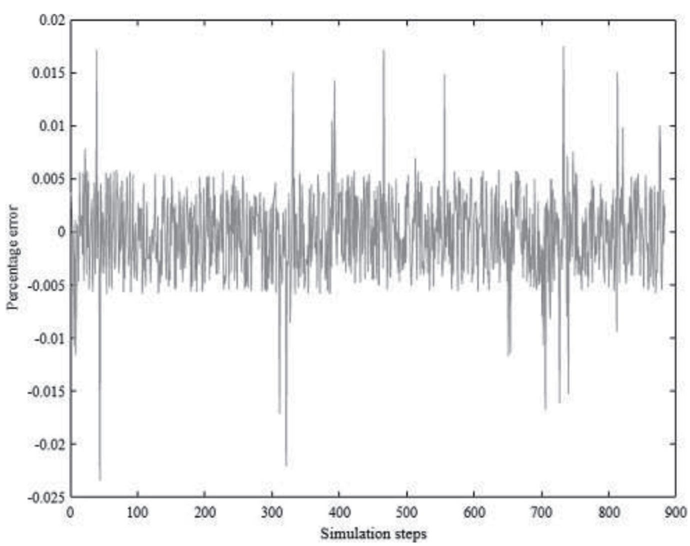

Figure 2 – Difference between empirical and simulated price indexes, regular network
Source: own elaboration.

Figure 2 shows the difference between the empirical price index and the simulated price index generated with the parameters from Table 1. We see that the percentage error is small, but it does raise in both directions when the synthetic series overshoots or undershoots due to outliers. That is to be expected considering that the calibration algorithm is quite simple and outliers are a known problem in synthetic data generation as pointed out by Tucker et al. (2020). Figure 3 shows price level and output evolution, where it does not converge to a perfectly elegant $P = Y = 1$ such as Ball and Romer (1989): price level fluctuates between 1 and 1.01

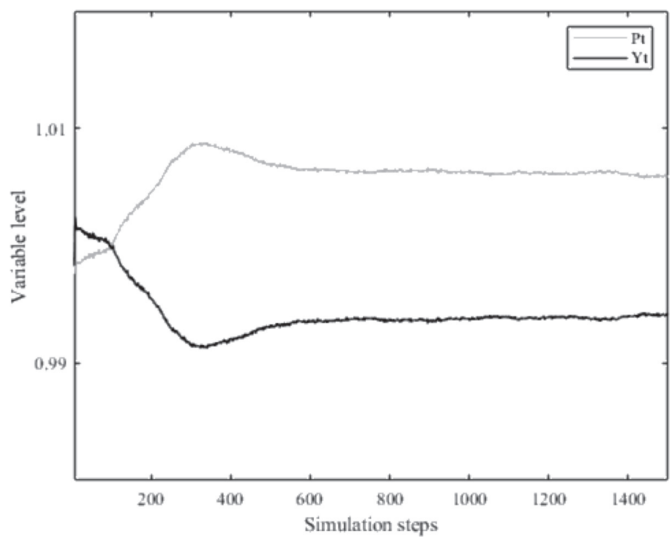

Figure 3 – Price and output level evolution, regular network
Source: own elaboration.

and output level fluctuates between 0.99 and 1 after step 500. As seen in Figure 4, price level variance is very low and converges to zero between steps 500 and 1,000, correlated with the dying out of the deflationary strategy (see Figure 5). For variance plots we have excluded the first 100 observations due to an initial overshooting while agents are still learning.

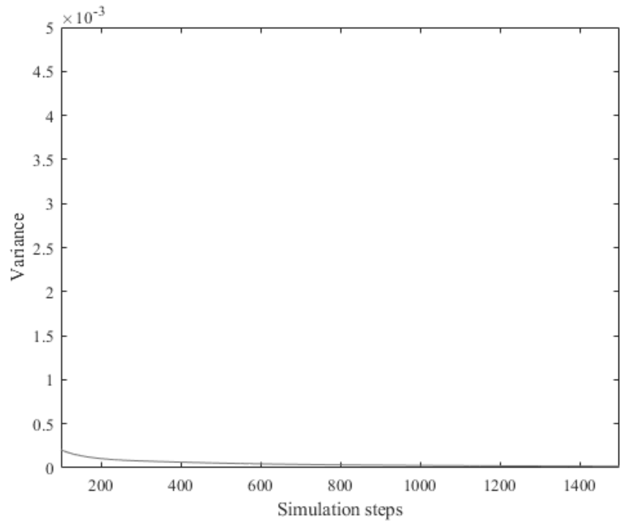

Figure 4 – Price level variance, regular network
Source: own elaboration.

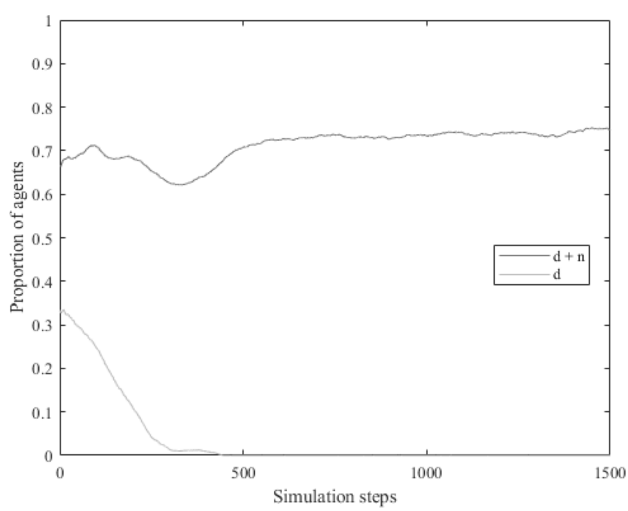

Figure 5 – Strategy distribution evolution, regular network
Source: own elaboration.

Next is the strategy distribution evolution (Figure 5), which should be read as follows: below the d line we have the proportion of deflationary agents at the present step and below the *d + n* line we have the proportion of deflationary and neutral agents, therefore the gap between the d and $d + n$ lines represents the proportion of neutral agents. Everything above $d + n$ is the proportion of inflationary agents. Here there is no convergence towards one strategy even as price level and output stabilise, with the inflationary strategy coexisting with the neutral strategy in the long run.

4.2. Calibration of the proposed ABM: a price-setting game on a complex network

Now we will compare a complex network's results with the results previously shown. Running the optimisation algorithm, our initial guess was the set from Table 1 along with a value of $p \approx 0.1$, which is the standard rewiring probability for small-world networks. After roughly 176,000 steps – that is, 200 runs of our data set – the optimisation algorithm found a combination of parameters. The results can be found in Table 2.

The closest result to the initial guess is ϕ, whose value, much closer to one than to zero, shows that in choosing their optimal price, agents strongly value their expected price rather than changes in money supply. This shows that the elasticity towards the money supply and price level do not seem to be dependent on the network structure. The β value, which was very close to the initial guess, shows that there is a relatively high weight from deterministic incentives on agents' choices. Hence, each agent's individual decision will not be taken very randomly.

Table 2 – Calibrated values for the main model

Parameter	Value
α	2.0288
β	2.0279
ϕ	0.7986
J	2.3246
p	0.5267

Source: own elaboration.

In turn, α is much higher than the initial guess and shows there is a significant incentive from private behaviour in setting prices, much more than in the benchmark model where $p = 0$. Which means that in a complex network, the utility associated to setting the price in the same direction as the general price level is more important than in the original model.

J presents a moderate value, lower than shown by the model with a regular network. This one is interesting considering that agents are more connected but consistent with a higher value of α, and it might actually be an effect of the network structure, which is the most important change in the model. In turn, p goes well beyond the expectation of a small-world network ($p \approx 0.1$).

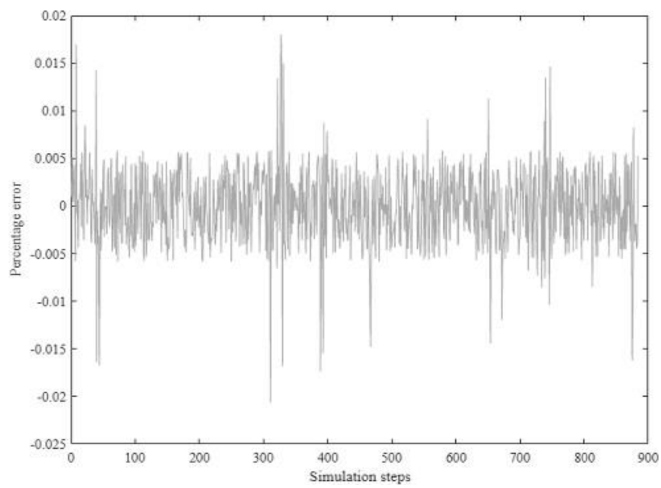

Figure 6 – Difference between empirical and simulated price indexes, complex network

Source: own elaboration.

As done beforehand, Figure 6 shows the difference between the empirical price series and the synthetic series generated by the set of parameters from Table 2. Just like Figure 2, it does feature some larger errors when trying to keep up with outliers, but like the regular network version, errors are quite small on average. Thus, we can follow with our analysis.

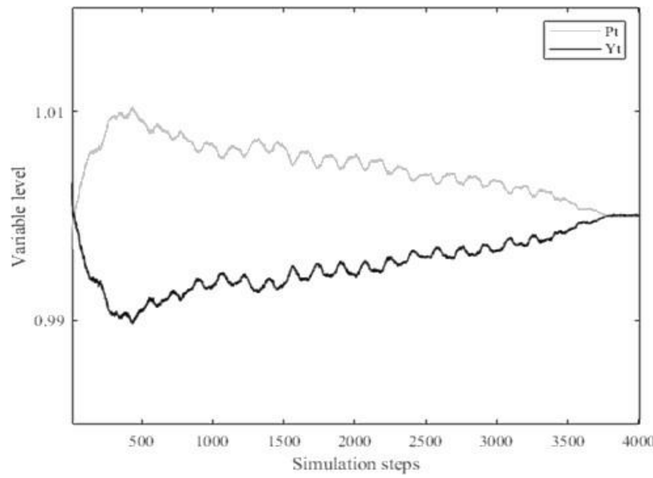

Figure 7 – Price and output level evolution, complex network

Source: own elaboration.

Our first analysis (Figure 7) was on the $P = Y = 1$ equilibrium. We had reproduced this result closely but not perfectly on the benchmark model. Now there is a perfect, inverse correlation between P and Y and, between steps 3,500 and 4,000, the economy stabilises on $P = Y = 1$, with a higher precision than in Figure 3. In turn, the variance (Figure 8) is larger when compared to the benchmark (Figure 4), as it takes as long as step 2,500 to become equal to the variance from the benchmark but does not get as close to zero afterwards. This was somewhat expected, considering there is now a greater degree of randomness.

4.3. Emergent properties

The previous section highlighted some differences between the regular network model and our main complex network model, most interesting being the long-run survival of inflationary expectations. Now we are faced with the most striking difference so far, which is an emergence of the neutral strategy as the dominant one, as shown in Figure 9. Due to the randomness, none of the three strategies will truly die out, but the deflationary and inflationary together end up fluctuating around 1% as we reach $P = Y = 1$, which might as well be considered an extinction. We will run several tests by stressing different parts of the model and checking out what changes happen, if any. To better understand how starting conditions can create very different emergent properties, we will run three tests. They will assess the effect of all agents starting with the same strategy, meaning all 10,000 of them will start being either deflationary, neutral or inflationary.

The first test (Figure 10) shows what happens when all agents are deflationary in the first step, that is, $\sigma_{i,1} = -1$ for any $i = 1, 2, \ldots, 10^4$. We have already understood this from previous results, but this is another evidence that the deflationary strategy is indeed the weakest. On step 2 almost all agents already move away from it, splitting between the neutral and inflationary strategies. Afterwards, we see the neutral strategy's climb towards the same spot as shown by the simulation with balanced starting conditions (Figure 9). It does crush the inflationary strategy much faster, amassing a 98% fraction around step 600, which is around 3,000 steps earlier.

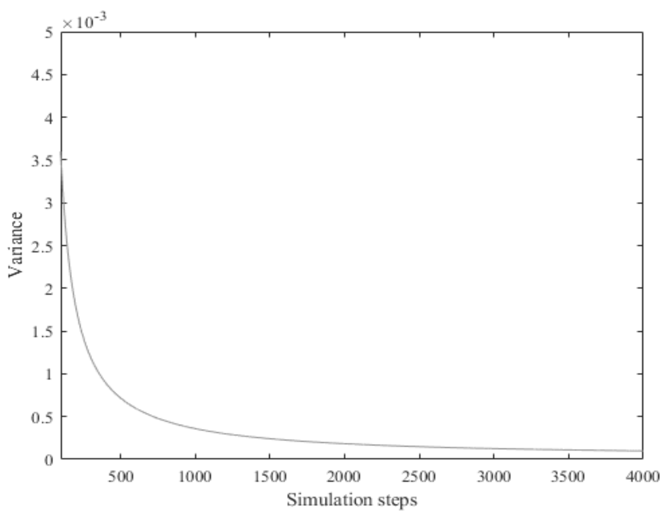

Figure 8 – Price level variance, complex network
Source: own elaboration.

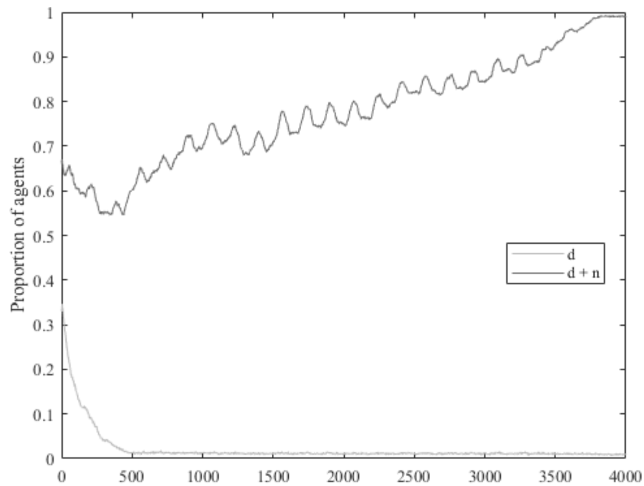

Figure 9 – Strategy distribution evolution, complex network
Source: own elaboration.

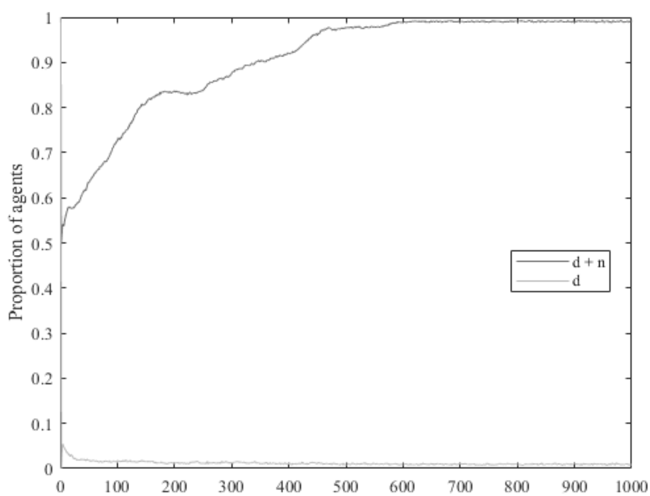

Figure 10 – Strategy distribution evolution, all agents starting as deflationary
Source: own elaboration.

The second test (Figure 11) has all agents starting as neutral, that is, $\sigma_{i,1} = 0$ for any $i = 1, 2, \ldots, 10^4$. This one shows a very surprising result, as in step 2 the neutral strategy – dominant in all simulations so far – is switched out by a majority of agents. Between step 200 and 400 it is already very close to extinction. Then the deflationary strategy slowly dies out and by step 1,400 the inflationary strategy is already the choice of 99% of the agents. With this result, the deflationary strategy is cemented as the weakest one, as it is never the dominant strategy by the end.

Ultimately, the third test (Figure 12) features all agents starting with the inflationary strategy, that is, $\sigma_{i,1} = 1$ for any $i = 1, 2, \ldots, 10^4$. Now it is the inflationary strategy that is switched out by almost all agents in the second step, followed by the fastest of the near-extinctions in all of the results, with the neutral strategy establishing its firm dominance by step 500. Looking at Figures 10 and 12, they look like imperfect mirrors of each other, with the universal starting strategy getting close to extinction during the first steps and the opposite strategy being dominated by the neutral one and being asymptotically extinct around step 600.

Something common between the tests is that by step 2, almost all agents discard the starting strategy – similar tests on the regular network model show that when all agents start on a single strategy, they never change.

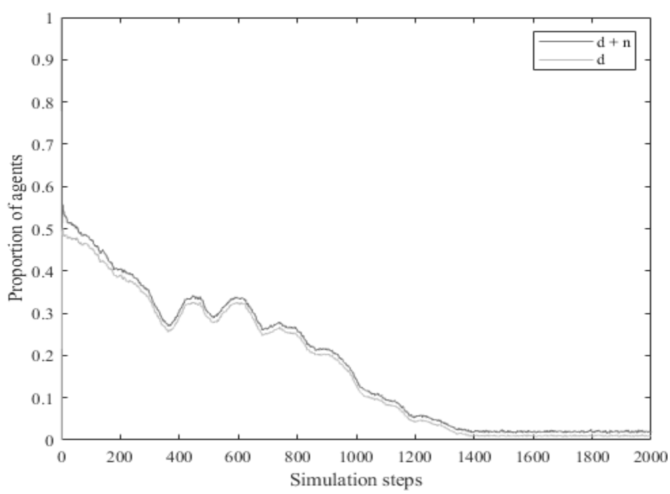

Figure 11 – Strategy distribution evolution, all agents starting as neutral
Source: own elaboration.

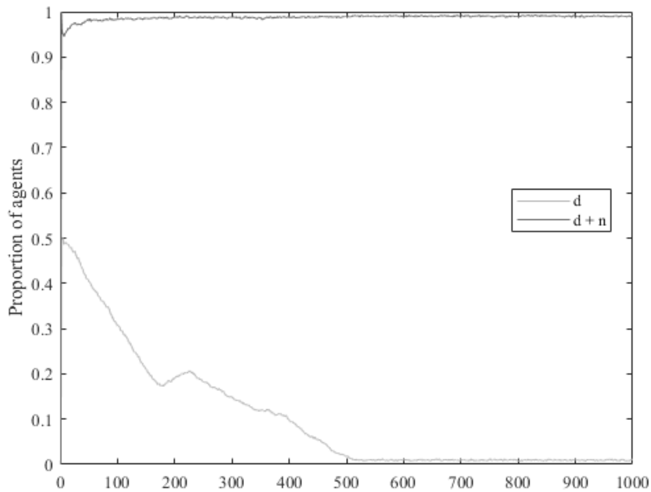

Figure 12 – Strategy distribution evolution, all agents starting as inflationary

Source: own elaboration.

This reinforces the strength of the neutral strategy, as the only test where it is not the winning strategy is the one where it starts as the only strategy, and we have seen that it means the strategy is doomed to die out when that happens. We believe that this is caused by a bigger value of α, giving more weight to whether agents get the price change right compared to the global price level or not, and until settling near P = 1 it is quite volatile (check Figures 3 and 7), so they will certainly choose the wrong strategy at one of the early steps.

After all of them change to another strategy, there are big incentives in continuing with it, as the intensity of choice value (β = 2.0288) being strictly positive prevents them from doing many random changes and the value of parameter J = 2.3246 prevents them from intensely deviating from their neighbours. Therefore, it makes sense that the starting strategy is brought to the brink of extinction very quickly.

Furthermore, we can also test how the model's parameters affect strategy distributions. To do so, we create vectors with 101 values of three parameters of interest, α/J, β and p. For each parameter, we take the calibrated value as the central value and generate 50 equidistant values to the left and the right of the central value, with the lower bound being zero and the upper bound being double the central value. Thus, each of these tests is comprised by 100 runs of the model.

We run simulations with $t = 1, 000$ and discard the first 100 observations to avoid biases due to the very high degree of randomness before agents start settling into a distribution pattern that will be more representative of the overall trend for that specific parameter value. After doing that, we calculate the mean of the remaining 900.

The first test evaluates how changing the α/J ratio impacts the frequency distribution of strategies across producers, as this ratio captures the relative weight of private incentives with respect to social incentives. This means that agents will give more importance to guessing the prices right in lieu of following their neighbourhood along. The simplest way to do so is keeping $J = 2.3246$ as it was calibrated and manipulating the value of α. Therefore, the interval starts at zero and is incremented by 0.0174 each time, thus closing at the value $\alpha/J = 1.74$.

Figure 13 shows the results found, where we observe less agents choosing deflationary and inflationary expectations as the α/J ratio gets larger. Therefore, as the deterministic private utility's relative weight rises, a larger fraction of agents go with the neutral strategy, with the average proportion of agents choosing it getting as large as 95%. That means when network effects are relatively weak (i.e. when the ratio α/J is relatively high), the neutral strategy is the one that provides agents with the largest utility gains. However, for low values of α/J, which gives a small relative weight to private incentives, the neutral strategy is indeed more vulnerable. For several values of $\alpha/J < 1$, the inflationary strategy is able to hold a majority, and for some values around 0.1, the neutral strategy in fact is going extinct.

Next, we test for values of β, the parameter that determines the intensity of choice, accounting for bounded rationality in the model. If values of β are low, we expect the proportions to remain near 33% *ad infinitum*, because choices are random. Ath the same time, larger values mean that agents will gravitate towards the strategy with the largest utility gains.

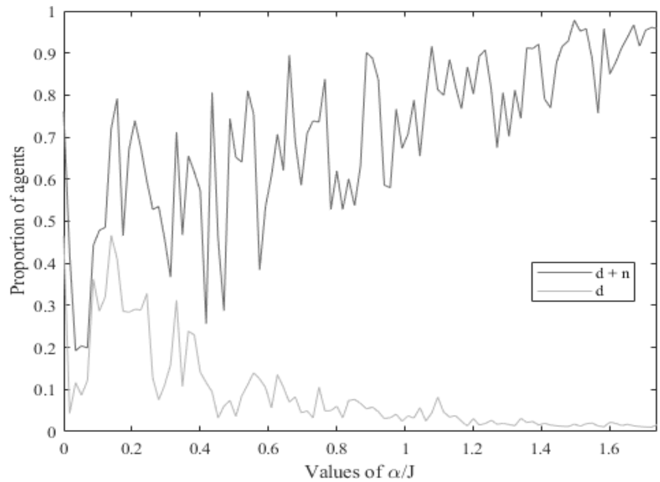

Figure 13 – Average strategy distribution evolution as a function of α/J
Source: own elaboration.

That means we expect that as agents become more rational – that is, with β raising – they will go with the strategy that has been showing the best performance so far, which is neutrality. Again, the interval starts at zero and ends at β = 4.045 (twice the calibrated value) with increments of 0.04005 each time.

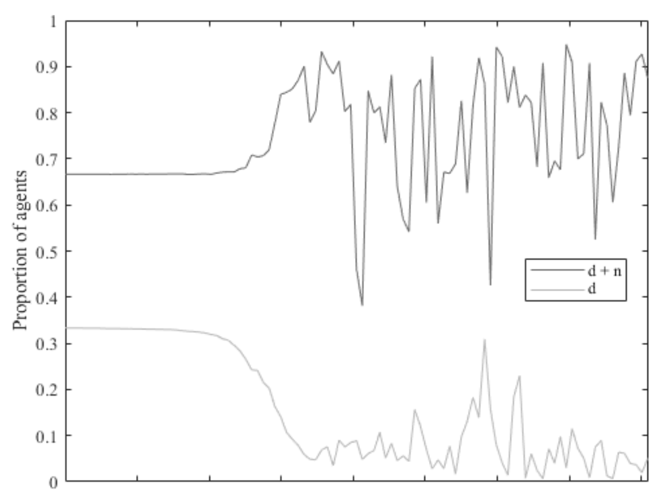

Figure 14 – Average strategy distribution evolution as a function of β

Source: own elaboration.

Figure 14 presents our results: for low values of β our test reflects what was mentioned before, with values of $\beta < 1$ leading to the three strategies coexisting equally in the long run, even though there is a clear distinction in utility gains from picking between them. Nevertheless, as predicted by Hommes (1997), when β rises agents pick with higher probability the strategy with relatively better performance in gaining utility, which is the neutral strategy, as the furthest value from the origin shows the largest proportion of neutral agents. The path is not the smoothest, but it does show a trend of agents moving towards the neutral strategy as decisions become more deterministic.

The last test is for values of rewiring probability, p, with results shown on Figure 15. If $p = 0$ then we have a regular network with no shortcuts, and if $p = 1$, a totally random network. We account for all values between 0 and 1 with a 0.01 increment. Unlike the previous results, there is not a clear trend associated with raises in the rewiring probability, however the neutral strategy is established as a clear favourite for most values of p. It does remind us again that the neutral strategy is, on average, much more likely to emerge as the clearly dominant one, although for some values of p it faces decent challenge from the inflationary strategy.

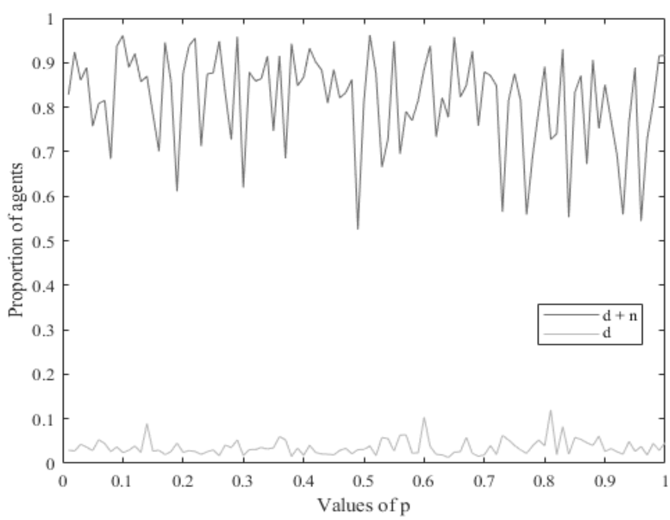

Figure 15 – Average strategy distribution evolution as a function of p
Source: own elaboration.

5. Concluding remarks

Lately, ABMs have presented themselves as a versatile, bottom-up approach aided by the advances in complexity and network sciences. These tools can better explain and translate the increasing complexity of markets in general. As such, they are able to tackle issues that can be hard to include in mainstream models, e.g. network effects and heterogeneous expectations.

Our main objective was to assess how the relaxation of some hypotheses affected the results of a model proposed by Ball and Romer (1989) on the theme of price setting. To do so, we have adapted a new Keynesian model into an agent-based computational model featuring heterogeneous agents with interactive expectations, bounded rationality and parameters calibrated by numerical methods. Using a regular network version as the benchmark, we also have added a rewiring probability to check whether it improved or not the model.

Each agent could, on each period, go with deflationary, neutral or inflationary expectations. The calibrated parameters presented a higher emphasis on the relative weight of private utility with respect to network effects compared to the regular network model. In the same vein, it showed a larger weight on deterministic utility and a higher elasticity towards one's expected price level compared to the money supply. The rewiring probability presented a rather high value showing that, when setting prices, agents may be highly influenced by distant nodes in the network.

While the regular network was unable to show the emergence of the neutral strategy as the dominant one, by adding the rewiring probability we saw that happening. Some tests were done to observe other emergent properties, with the first one being a change to the starting conditions. By making all agents start with each one of the three strategies, we have seen that by step two of the simulation all of them change strategies, with the starting one going extinct and the remaining two vying for dominance.

These tests regarding the sensitivity to initial conditions show the neutral strategy is the strongest, and the inflationary can be said to be second place, as it was able to crowd out the deflationary strategy when all agents started as neutral (which guaranteed it would die out soon, for reasons not entirely clear to us).

We also stressed all parameters to check their effects on strategies. When α grows, the neutral strategy becomes more dominant, showing it is stronger when agents look only at their accuracy when comparing with the global price level. Likewise, when β grows, meaning agents become more rational, they gravitate towards the neutral strategy. However, changes in p do not show any particular trend when running the gamut from zero to one.

It is also most interesting that, even though we have relaxed several hypotheses of the original new Keynesian model, the conclusions are mostly unchanged: agents reach $P = Y = 1$, reinforcing that heterogeneous expectations and the fear of getting the price wrong compared to one's peers (thus, losing out on profits) are factors that can contribute to price stickiness. Still, as Guerrero and Axtell (2011) point out, by stressing mainstream models and reaching similar conclusions, we can be more appreciative of the models that do hold up even under heavy scrutiny.

In fact, our main takeaway from the comparison between regular and complex network results is that some degree of randomness was needed to perfectly replicate the symmetric Nash equilibrium from Ball and Romer (1989), which went hand-in-hand with the emergence of the neutral strategy as the strictly dominant one. Thus, complex networks might be better-suited to represent price setting phenomena, which makes sense as economic agents are not influenced only by neighbours.

As this is a quite new research agenda, our hand was forced in some decisions, such as the price index and calibration method choices; still, we ran a high amount of simulations and believe our results to be highly trustworthy, even if not fully optimised. Some suggestions for future research on the same theme would be: adding monetary policy; using a preferential attachments network instead of a small-world one; increasing neighbourhood size; using other arrangements instead of square lattice; changing the calibration method and/or price index.

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