Research article

Global shocks and trade response-terms of trade, J-curve and the Marshall-Lerner condition: evidence from Brazil♦

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Abstract

The traditional terms of trade (TOT) shock analysis may be an unrealistic experiment, especially for a small open economy where the state of the world economy endogenously determines TOT. The dynamic relation between TOT and the trade balance needs to consider the nature of the global shock that jointly moves price and income, globally and domestically. We estimate Bayesian global SVARs for Brazil using a structure that easily communicates to traditional DSGE models to evaluate the presence of the J-curve and the Marshall-Lerner condition (MLC) following innovations we claim to resemble world supply and demand shocks. We do not encounter a J-curve for aggregate trade nor for trade in fuel, and capital and consumption goods. MLC is not verified only for trade in capital goods, but even in the cases where MLC is present, we observe that TOT and the volume exported are not correlated as expected, since improvement in the volume exported happens when TOT appreciates. Our results suggest that income effect plays the most prominent role for the dynamics of the trade balance and for the validity of the MLC, dominating the TOT effect.

Keywords

J-curve; Marshall-Lerner condition; Terms of trade; Global shocks; SVAR.

Choques globais e respostas de comércio - termos de troca, curva J e condição de Marshall-Lerner: evidências para o Brasil

Resumo

A tradicional análise para avaliar impactos de choques nos termos de troca (TOT) pode ser um experimento irreal, especialmente para pequenas economias abertas em que TOT é endogenamente determinado pelo estado da economia mundial. A relação dinâmica entre TOT

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e balança comercial deve assim considerar a natureza dos choques que determinam conjuntamente preços e renda, doméstica e internacionalmente. Estimamos SVAR global bayesiano, cuja estrutura permite fácil comunicação com tradicionais modelos de equilíbrio geral dinâmico estocástico, para avaliar a presença da curva J e da condição de Marshall-Lerner (MLC) para a economia brasileira na presença de inovações que alegamos assemelhar-se a choques de oferta e demanda globais. Não foi encontrada a curva J para o comércio agregado e nem para comércio de combustíveis, de bens de capital e de consumo. Apesar da MLC não ser verificada apenas para bens de capital, o volume exportado e TOT não se correlacionam como esperado por teorias tradicionais, pois melhorias no primeiro ocorrem quando o segundo aprecia-se. Os resultados sugerem que o efeito renda é o principal determinante da dinâmica da balança comercial e para a validade da MLC, dominando assim o efeito dos termos de troca.

Palavras-chave
Curva J; Condição de Marshall-Lerner; Termos de troca; Choques globais; SVAR.

Classificação JEL
C32, E20, F44

1. Introduction

The J-curve and the Marshall-Lerner condition (MLC) are benchmark concepts in international economics that relate the response of the trade balance (TB) to changes in the terms of trade (and/or real exchange rate)[1]. MLC establishes conditions on the elasticity of the volume imported and exported that guarantee that a depreciation (appreciation) improves (worsens) the trade balance. The J-curve describes the dynamics of TB in response to shifts in the terms of trade (*TOT*). Accordingly, a depreciation would first deteriorate the TB before improving it, because more expensive imports would initially dominate the balance result.

We investigate if the J-curve and the MLC are observed in Brazil, a net commodity exporter emerging country, after exposing its economy to global demand and supply shocks. We consider this a more proper way to study the topic, since oscillations in *TOT* are mainly determined by the state of the international economy, which affects not only prices, but also income, demand, and supply, globally and domestically. In this sense, our analysis distinguishes itself from most empirical works in the field that tend to consider innovations directly applied to the terms of trade (or in

[1] There are papers that consider these concepts with respect to terms of trade and others to real exchange rate.

the real exchange rate), an experiment we consider unrealistic since export and import price indexes respond endogenously to economic meaningful global shocks.

We follow Blanchard (1989) and think of global supply and demand innovations as described in standard textbook examples: an adverse (a good) supply shock is associated with a price increase (decrease) and a drop (rise) in income, while an adverse (a good) demand shock is associated with smaller (higher) prices and income[2]. These shocks are identified from a structural (global) vector autoregression (VAR) containing domestic and international blocks. Our analyses are based on impulse response functions and the models are estimated using Bayesian techniques specially designed by Cushman and Zha (1997) and Zha (1999) to accommodate a complete block recursive restriction, a necessity when modeling the international influence on a small open economy.

We gain insights on the operating channels determining the trade balance by also including the domestic GDP, household consumption and investment in the VAR, allowing comparisons to the general equilibrium models of Backus, Kehoe and Kydland (1994) and Senhadji (1998). A second VAR including volumes imported and exported provides additional perspectives on the channels and allows studying the MLC condition. To be aligned with Backus, Kehoe and Kydland (1994), we adapt the model of Boyd, Caporale and Smith (2001) to consider price incentives arising from oscillations in TOT instead of real exchange rate (*RER*) and show how to evaluate the elasticities supporting the MLC from the impulse response functions.

Our results indicate improvement of the Brazilian *TOT* after a positive world demand shock and a negative world supply shock. The response of the aggregate *TB* reveals the absence of the J-curve after exposing the Brazilian economy to both innovations, the same happening when restricting the exercise to fuel, and capital and consumption goods.

Except for trade in capital goods, the MLC is mostly verified: *TB* improves (worsens) as *TOT* depreciate (appreciate). However, the volume exported increases (falls) following an improvement (worsening) in *TOT*, opposing the standard assumption behind the construction of MLC. We even

2 Bekaert, Engstrom and Ermolov (2021) and Ermolov (2022) are recent examples that adopt similar perspective to identify aggregate supply and demand shocks.

verify improvement in the volume exported of capital and consumption goods after an appreciation of *TOT*, distancing the Brazilian experience from concerns regarding the Dutch disease, at least in the business cycle frequency. The apparent contradiction between the validity of the MLC with an opposite response of the volume exported happens because the income effect, captured by the responses of world and domestic GDPs, is a stronger force determining the volumes traded than the price effect, something that is absent from the partial equilibrium structure that is normally behind the models that derive the MLC.

The encountered relation between *TOT* and trade balance is aligned with the findings of Backus, Kehoe and Kydland (1994) and Senhadji (1998) who recognize the endogenous status of *TOT*, but opposes the theoretical predictions of Harberger-Laursen-Metzler and Obstfeld-Razin-Svensson effects[3], and the empirical findings of works that treat *TOT* as an exogenous variable from the structure of the world economy (Schmitt-Grohé and Uribe (2018), for instance).

1.1. Literature

Backus, Kehoe and Kydland (1994) and Senhadji (1998) develop open economy models where movements in *TOT* and *TB* are determined endogenously in a general equilibrium environment following economic meaningful shocks. Backus, Kehoe and Kydland (1994) analyze responses to two domestic shocks: supply (productivity) and demand (government expenditure). Since they build a two-country model, these domestic shocks are also global. Senhadji (1998) extends the model to a small open economy environment to search for the J-curve, but the absence of a proper general equilibrium structure at the world level does not allow verifying the consequences of a global supply shock. Instead, the author focuses on the impact of a domestic productivity shock, repeating the exercise of Backus, Kehoe and Kydland (1994), and of a global demand shock. Despite these differences, both works reach similar conclusions: i) a local positive productivity shock depreciates *TOT* and produces a J-curve, with the *TB* initially worsening before improving; and ii) a positive global demand shock appreciates *TOT* and deteriorates the *TB*, but without producing a J-curve.

3 See Uribe and Schmitt-Grohé (2017), chapter 7, for a discussion.

Despite the theoretical advancement provided by Backus, Kehoe and Kydland (1994) and Senhadji (1998), empirical works that followed aimed at evaluating the J-curve and the MLC do not consider a general equilibrium perspective where economic meaningful global shocks are the primary source of oscillation in *TOT* and income. This pattern is exposed in the extensive and detailed literature review conducted by Bahmani-Oskooee and Ratha (2004) and Bahmani-Oskooee and Hegerty (2010). Apart from the works of Backus, Kehoe and Kydland (1994) and Senhadji (1998), a large portion of the empirical papers cited by these reviews analyze the J-curve after exposing the system to a *real exchange rate shock*[4]. Another common practice is to rely on VECM to verify short and long run coefficients of regressions of the trade balance on real exchange rate (*RER*) or *TOT*, but without addressing the influence of global shocks that can jointly affect the main covariate of interest and the regression's error, which may lead to endogeneity. The long run analysis verifies the validity of the MLC by checking the sign of the coefficient linking the RER with the trade balance conditioned on cointegration. Conditioned on the validity of the MLC, the J-curve is the focus of the short run analysis, which is mostly verified through impulse response analysis. To the best of our knowledge, the works for Brazil applied to similar product aggregates as ours fall in these strands (Moura and Silva (2005), Reis Gomes and Paz (2005), Arruda, Castelar and Martins (2019), Bahmani-Oskooee, Arize and Kalu (2022), and Arruda, Brito and Castelar (2022)). From the econometric perspective, Rocha, Magalhães and Brilhante (2024) is closer to our approach, as they also rely on a Bayesian VAR with variables in level to check for the MLC and the J-curve, but still entertain the *real exchange rate shock*.[5]

Another common practice in the literature is to consider trade between countries (Bahmani-Oskooee, Arize and Kalu (2022)), which may be informative on a certain level, especially when changes in RER correspond to reactions to local policies that are restricted to a small open economy (like a local adverse sovereign risk shock that devalues the nominal exchange rate), but seems limited when it comes to aggregate global shocks that affect most countries. To the extent that shifts in *RER* and in the demand for imports are in large part determined by global events that

4 Even Ali and Anwar (2018) use their small open economy DSGE model to evaluate the presence of a J-curve by exposing the economy to a *TOT* shock.

5 de Azevedo *et al.*(2024) also rely on VECM to verify the responses after a *real exchange rate shock*, but their analyses are restricted to trade in agriculture product, which we do not analyze. Arruda and Martins (2020) use VECM to check for the presence of a J-curve in local regional trade after a *RER shock*.

affect countries disproportionately, focusing on bilateral trade does not provide a broader picture that matters most for understanding aggregate movements, like the impact on the entire trade balance. This problem may be aggravated when both countries have gone through structural changes that alter the importance of the main trade partners, since this potentially modifies the global forces acting in the determination of nominal and real exchange rates.[6]

Another pitfall related to exercises based on bilateral trade is that it is not suitable for verifying the channels responsible for validating or not the J-curve and the MLC, something that becomes possible to approach when the model is estimated under a structure that recognizes the general equilibrium linkages at the world level and incorporates domestic macroeconomic aggregates, as in the DSGE models of Backus, Kehoe and Kydland (1994) and Senhadji (1998).

Overall, our paper is related to the extensive literature focused on analyzing the relation between *TB* and *TOT*, but an important distinction is our interest in evaluating the reactions to innovations in world supply and demand, since they should be the main drivers of variations in *TOT* over the business cycle. For instance, Kose (2002), Drechsel and Tenreyro (2018), and Fernández, González and Rodriguez (2018) consider shocks in the international prices determining *TOT*, but their innovations do not have a proper economic interpretation (supply or demand), while Mendoza (1995), Schmitt-Grohé and Uribe (2018) and Fernández, Schmitt-Grohé and Uribe (2017) treat *TOT* as an exogenous variable subject to “terms of trade shock”.[7]

In another direction, Ferreira and Valério (2022) show the relevance of considering a general equilibrium perspective in a VAR to identify global economic meaningful shocks to analyze their impact on the real and monetary side of the following commodity exporters: Brazil, Chile, Colombia,

[6] In 2001, the year China joined the WTO, 3.3% of the Brazilian merchandise exports were directed to China and 24.7% to the USA; in 2019, exports to China represented 28.7% and to USA 13.5%. The import figures went through similar change: 2.4% of the Brazilian merchandise imports came from China in 2001 while 22.7% were originated in the USA. In 2019, China contributed with 19.4% and USA with 19.0%.

[7] Fernández, González and Rodriguez (2018) go a step further and evaluate the impact of innovations arising at the world level, but the economic interpretation of these shocks is harmed by the fact that they do not link all international variables in a full general equilibrium structure. Ferreira and Valério (2022) conduct a detailed discussion about the methodology employed by Fernández, González and Rodriguez (2018).

and Peru[8]. Charnavoki and Dolado (2012) pursue a similar approach to study the influence of international shocks on Canada, a commodity exporter rich economy, and revisit several topics in the international economics literature related to terms of trade. We follow one of their analyses and discuss the presence of the Dutch disease by focusing on the export response of capital and consumption goods.

The findings of Ferreira and Valério (2022) are of special interest for the MLC and J-curve literature that has been developed for Brazil, mainly built on the relation between *TB* and *RER*, instead of *TOT*[9]. One could argue, for instance, that relying on *RER* alleviates the need to consider global shocks based on a supposition that the *RER* is mostly determined domestically. However, Ferreira and Valério (2022) estimate that 70-82% of the variance of the forecasting errors of the Brazilian nominal exchange rate (from 2002 to 2019) is accounted by world shocks (supply, demand, and uncertainty), while 12-20% is due to domestic sovereign risk shocks. These numbers reinforce the need to consider a general equilibrium at the global level to analyze MLC and J-curve no matter if considering *TOT* or *RER*. It is worth emphasizing once again that we choose to work with *TOT* to remain closer to the general equilibrium models of Backus, Kehoe and Kydland (1994) and Senhadji (1998).

1.2. The structure of the Brazilian trade in goods

To contextualize our analyses, it is interesting to have a broad picture of the structure of the Brazilian trade in goods. Table 1 shows that 65.8% of the country's export in 2019 were concentrated in intermediate goods that were responsible for US$41.6 billion of the US$48.0 billion total trade surplus. Table 2 shows that the 8 major exporting products, which together represented 56.1% of total exports, are all commodities from a variety of sectors: food, oil, and basic mineral[10]. This diversification fur-

8 Ferreira and Valério (2022) analyze how macroeconomic variables of Brazil, Chile, Colombia, and Peru react after global shocks in demand, supply, and uncertainty.

9 Moura and Silva (2005), Reis Gomes and Paz (2005), Arruda, Castelar and Martins (2019), Bahmani-Oskooee, Arize and Kalu (2022), Arruda, Brito and Castelar (2022), Azevedo *et al.* (2024), Rocha, Magalhães and Brilhante (2024) conduct their analyses for Brazil using *RER*.

10 Although the data used to construct Tables 1 and 2 come from the Ministry of the Economy, they present a small difference we were unable to identify the reason. For instance, while total exports equal US$ 225.4 billion in Table 1, the primary information used to build Table 2 presents total exports of US$ 221.1 billion. This small difference does not modify the big picture obtained from both tables.

ther justifies our interest in analyzing the impact of aggregate world shocks instead of considering events restricted to a single commodity. The trade structure also places Brazil as a price taker for its exports and imports, which asks for an adequate modelling of the world economy to analyze the response of local trade variables.

The rest of the paper proceeds as follows: in section 2 we present the data and the methodology, including the SVAR and the economic model adapted from Boyd, Caporale and Smith (2001). Section 3 presents the main results for aggregate trade variables, while section 4 focuses exclusively on disaggregated goods: capital goods, consumption goods, and fuel. In section 5 we conduct a discussion comparing our findings to others in the literature, also highlighting differences in methodologies. Section 6 concludes.

Table 1 - International Trade - Brazil (2019)

Category	Exports (US$ billions)	% of exports	Imports (US$ billions)	% of imports	Net trade (US$ billions)
Capital goods	16.2	7.2	25.2	14.2	-9.0
Intermediate goods	148.3	65.8	106.7	60.2	41.6
Consumption goods	30.8	13.7	24.6	13.9	6.2
Fuel	30.1	13.3	20.7	11.7	9.4
TOTAL	225.4	100	177.3	100	48.0

Source: IPEA - www.ipeadata.gov.br

Table 2 – Main exporting products: Brazil (2019)

Product	Value (US$ Billions)	% of Total Exports
1. Soybean complex	32.3	14.6
2. Petroleum (crude and fuel)	30.1	13.6
3. Iron Ore	22.7	10.3
4. Meat (bovine, poultry, hog)	14.5	6.6
5. Sulfate, chemical, wood pulp	7.5	3.4
6. Maize	7.3	3.3
7. Sugar	5.2	2.3
8. Coffee	4.6	2.1
Total		56.1

Source: Brazilian Ministry of Economy - https://balanca.economia.gov.br/balanca/SH/ISIC$_C$UCI.xlsx

2. Methodology and Data

The analyses are conducted by simulating impulse response functions from a structural VAR (SVAR) model. The set up is designed to accomplish the fact we are modeling world shocks that affect a small open economy, which requires imposing a bloc exogeneity structure for the international variables. Following Cushman and Zha (1997), consider the following SVAR representation:

$$A(L)y(t) = \varepsilon(t) \tag{1}$$

where $y(t)$ is a $n \times 1$ vector of variables, $A(L)$ is a $n \times n$ matrix of coefficients with L being the lag operator, and $\varepsilon(t)$ a $n \times 1$ vector of structural innovations. We can partition the vectors and the matrix $A(L)$ to represent the dynamics of the system in terms of two groups of variables, $y_1(t)$ and $y_2(t)$:

$$\mathbf{y}(\mathbf{t}) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \mathbf{A}(\mathbf{L}) = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix}, \varepsilon(\mathbf{t}) = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \tag{2}$$

The dimension of A_{11} is $n_1 \times n_1$, A_{12} is $n_1 \times n_2$, A_{21} is $n_2 \times n_1$, A_{22} is $n_2 \times n_2$; $y_1(t)$ and $\varepsilon_1(t)$ are $n_1 \times 1$; $y_2(t)$ and $\varepsilon_2(t)$ are $n_2 \times 1$, where $n_1 + n_2 = n$.

The small open economy hypothesis is attained by imposing $A_{12} = 0$, so the Brazilian variables in y_2 do not affect, even recursively, those belonging to the global economy in y_1.

2.1. The world economy

We initially focus on the structure of the world economy represented by

$$\mathbf{y}_1(\mathbf{t}) = \begin{bmatrix} wgdp(t) \\ wp(t) \end{bmatrix}, \mathbf{A}_{11}(\mathbf{L}) = \begin{bmatrix} \alpha_{wgdp}^{wgdp}(L) & \alpha_{wp}^{wgdp}(L) \\ \alpha_{wgdp}^{wp}(L) & \alpha_{wp}^{wp}(L) \end{bmatrix}, \varepsilon(\mathbf{t}) = \begin{bmatrix} \varepsilon^{wd}(\mathrm{t}) \\ \varepsilon^{ws}(\mathrm{t}) \end{bmatrix}$$

$wgdp$ and wp correspond, respectively, to the logarithm of world GDP and to the logarithm of a world price index; ε^{wd} and ε^{ws} are innovations to the equation determining $wgdp$ and wp, which we will interpret as international demand and supply shocks after analyzing the behavior of the impulse response functions.

Following the VAR literature, the identification strategy assumes that $\varepsilon^{wd}(0)$ can impact the world price index contemporaneously, but $\varepsilon^{ws}(0)$ can affect global activity only with a period of delay. These hypotheses are captured by imposing $\alpha_{wp}^{wgdp}(0) = 0$.

2.2. The domestic economy

We verify the presence of the J-curve using a VAR that contains the same variables present in the general equilibrium model of Backus, Kehoe and Kydland (1994) and Senhadji (1998): terms of trade (*tot*); trade balance (*tb*); real investment (*inv*); real consumption (*cons*); and real domestic GDP (*gdp*), where small letters represent the natural logarithmic of their capital counterparts. It follows that $y_2' = [tot, tb, inv, cons, gdp]$. After considering global and domestic variables, the impact matrix $A_J(0)$ becomes.

$$\mathbf{A_J}(0) = \begin{matrix} wgdp \\ wp \\ tot \\ tb \\ inv \\ cons \\ gdp \end{matrix} \begin{bmatrix} \alpha_{wgdp}^{wgdp} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{wgdp}^{wp} & \alpha_{wp}^{wp} & 0 & 0 & 0 & 0 & 0 \\ \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} & 0 & 0 & 0 & 0 \\ \alpha_{4,1} & \alpha_{4,2} & \alpha_{4,3} & \alpha_{4,4} & \alpha_{4,5} & \alpha_{4,6} & \alpha_{4,7} \\ \alpha_{5,1} & \alpha_{5,2} & \alpha_{5,3} & \alpha_{5,4} & \alpha_{5,5} & \alpha_{5,6} & \alpha_{5,7} \\ \alpha_{6,1} & \alpha_{6,2} & \alpha_{6,3} & \alpha_{6,4} & \alpha_{6,5} & \alpha_{6,6} & \alpha_{6,7} \\ \alpha_{7,1} & \alpha_{7,2} & \alpha_{7,3} & \alpha_{7,4} & \alpha_{7,5} & \alpha_{7,6} & \alpha_{7,7} \end{bmatrix} \quad (3)$$

The index J in A_J is arbitrarily used to differentiate the impact matrix of this SVAR from the one used to evaluate the Marshall-Lerner condition, which we name A_M.

The zero restrictions in the lines $wgdp$ and wp repeat the identification scheme at the international level. The first two columns also show that world shocks may affect *tot* and the rest of local variables contemporaneously, which is accomplished by not imposing zero restriction on $\alpha_{i,j}$ for $j = 1,2$ and $i = 3,4,5,6,7$. This structure leaves *tot* endogenously determined by innovations at the world level.

It is not our intention to identify the other shocks, which is why we are not concerned in discussing the additional restrictions on $A_J(0)$ required for estimation purpose but that do not impact any of our results. For this reason, the superscript of the remaining innovations takes the name of the variable in the corresponding equation.

The small open economy hypothesis implies that local variables cannot influence the world economy variables, including the terms of trade[11]. This requires imposing restrictions in the coefficients of $A_J(L)$ for $L \geq 1$ as in matrix 4.

$$\mathbf{A_J(L)} = \begin{matrix} wgdp \\ wp \\ tot \\ tb \\ inv \\ cons \\ gdp \end{matrix} \begin{bmatrix} \varphi_{1,1} & \varphi_{1,2} & 0 & 0 & 0 & 0 & 0 \\ \varphi_{2,1} & \varphi_{2,2} & 0 & 0 & 0 & 0 & 0 \\ \varphi_{3,1} & \varphi_{3,2} & \varphi_{3,3} & 0 & 0 & 0 & 0 \\ \varphi_{4,1} & \varphi_{4,2} & \varphi_{4,3} & \varphi_{4,4} & \varphi_{4,5} & \varphi_{4,6} & \varphi_{4,7} \\ \varphi_{5,1} & \varphi_{5,2} & \varphi_{5,3} & \varphi_{5,4} & \varphi_{5,5} & \varphi_{5,6} & \varphi_{5,7} \\ \varphi_{6,1} & \varphi_{6,2} & \varphi_{6,3} & \varphi_{6,4} & \varphi_{6,5} & \varphi_{6,6} & \varphi_{6,7} \\ \varphi_{7,1} & \varphi_{7,2} & \varphi_{7,3} & \varphi_{7,4} & \varphi_{7,5} & \varphi_{7,6} & \varphi_{7,7} \end{bmatrix} \quad (4)$$

The zero restrictions in the first two rows clarify that only lags of $wgdp$ and wp can influence each other dynamically. Setting $\varphi_{3,1} \neq 0$ and $\varphi_{3,2} \neq 0$ explicit that $wgdp$ and wp can dynamically determine tot. Even though the prices responsible for determining tot are formed in the international market, the export and import contents of each country are related to structural factors of the small economy and should not affect world activity and aggregate price level, justifying $\varphi_{1,3} = \varphi_{2,3} = 0$. Similarly, since the evolution of the domestic variables does not influence the aggregate world price index, they should not affect tot at the business cycle frequency, which leads to $\varphi_{3,j} = 0$ for $j = 4,5,6,7$.

This SVAR structure permits evaluating the sign of the partial derivative $\frac{dtb}{dtot}$ to check whether the Marshall-Lerner condition (MLC) is present. But to gain insights regarding the channels behind the MLC, we estimate a second VAR with a structure that dialogues with the models commonly used to derive MLC. The new system incorporates the volume imported and exported, as their elasticities with respect to TOT form the MLC. We now briefly present the economic model normally used to derive MLC to show its connection to this second SVAR. We adapt the model presented by Boyd, Caporale and Smith (2001) to consider TOT of RER[12].

[11] This seems a reasonable hypothesis for our purpose, which is to evaluate responses at the business cycle frequency. This does not conflict with a local determination of the structural terms of trade, which is mostly related to a country's comparative advantage.

[12] When considering RER, one should consider domestic and international general price index, and

Let *TB* be defined as the ratio between the value exported and imported: $TB_t = \frac{P_t^X X_t}{P_t^M M_t}$, where P_t^X and P_t^M represent the price index of exports and imports, respectively, and X_t and M_t are the volume exported and imported, in this order. Taking the logarithm from both sides results in $tb_t = (x_t - m_t) - tot_t$, where $tb_t = log(TB)$, $x_t = log(X_t)$, $m_t = log(M_t)$, and $tot_t = p_t^m - p_t^x$. According to this last definition, an increase (worsening) in the terms of trade is paramount to a depreciation of the domestic currency while a decrease (improvement) parallels an appreciation.

To complete the model that gives rise to MLC, we need an export and an import function. The export function is represented by $x_t = \beta_x + \beta^{wgdp} wgdp_t + \eta_x tot_t$, and the import by $m_t = \beta_m + \beta^{gdp} gdp_t - \eta_m tot_t$. We follow Boyd, Caporale and Smith (2001) and assume that all coefficients are positive, so an increase in *tot* elevates x and reduces m. Higher world GDP increases domestic exports and higher domestic GDP elevates domestic imports. Because the variables are in logarithm, the coefficients represent elasticities. Replacing the export and the import functions in the trade balance equation results in

$$tb_t = (\beta_x - \beta_m) + \beta^{wgdp} wgdp_t - \beta^{gdp} gdp_t + (\eta_x + \eta_m - 1) tot_t \qquad (5)$$

MLC is the condition that guarantees that $\frac{dtb_t}{dtot_t} > 0$, which is satisfied when $(\eta_x + \eta_m - 1) > 0$. This implies, for instance, that a relative increase in the price of imports (increase in *tot*) produces a positive impact in the volume exported and a negative in the volume imported that are large enough to improve *TB* despite of higher import prices relative to exports. Putting in another way, the quantity effect surpasses the price effect.

This traditional approach to evaluate the presence of the MLC, however, misses important aspects related to the general equilibrium structure of the economy. Specifically, *TOT* is normally treated as an exogenous variable suitable to receive "terms of trade shocks". Under this perspective, the derivative $\frac{dtb_t}{dtot_t}$ also encompasses a notion of causality. However, the terms of trade, specially of a small open economy, is an endogenous variable that responds to world shocks that also provoke reactions in several domestic

when considering TOT we should consider the price index of export and import.

and international variables and the derivative $\frac{dtb_t}{dtot_t}$ does not allow a direct interpretation of causality between *TOT* and *TB*.

For all these, we evaluate the presence of the MLC using a SVAR that allows the identification of global structural innovations that endogenously affect *tot* and the other variables present in equation 5. Under this new SVAR, the vector of international variables y_1 remains the same as before. The vector of local variables is however different: $y_2' = [tot, x, m, tb, gdp]$. The impact matrix $A_M(0)$ is

$$\mathbf{A_M}(0) = \begin{matrix} wgdp \\ wp \\ tot \\ x \\ m \\ tb \\ gdp \end{matrix} \begin{bmatrix} \alpha_{wgdp}^{wgdp} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_{wgdp}^{wp} & \alpha_{wp}^{wp} & 0 & 0 & 0 & 0 & 0 \\ \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} & 0 & 0 & 0 & 0 \\ \alpha_{4,1} & \alpha_{4,2} & \alpha_{4,3} & \alpha_{4,4} & \alpha_{4,5} & \alpha_{4,6} & \alpha_{4,7} \\ \alpha_{5,1} & \alpha_{5,2} & \alpha_{5,3} & \alpha_{5,4} & \alpha_{5,5} & \alpha_{5,6} & \alpha_{5,7} \\ \alpha_{6,1} & \alpha_{6,2} & \alpha_{6,3} & \alpha_{6,4} & \alpha_{6,5} & \alpha_{6,6} & \alpha_{6,7} \\ \alpha_{7,1} & \alpha_{7,2} & \alpha_{7,3} & \alpha_{7,4} & \alpha_{7,5} & \alpha_{7,6} & \alpha_{7,7} \end{bmatrix} \quad (6)$$

The rationale for identifying the first SVAR applies to $A_M(0)$, the same holding for the restrictions imposed to the matrices containing the autoregressive coefficients $A_M(L)$, for $L \geq 1$:

$$\mathbf{A_M}(\mathbf{L}) = \begin{matrix} wgdp \\ wp \\ tot \\ x \\ m \\ tb \\ gdp \end{matrix} \begin{bmatrix} \varphi_{1,1} & \varphi_{1,2} & 0 & 0 & 0 & 0 & 0 \\ \varphi_{2,1} & \varphi_{2,2} & 0 & 0 & 0 & 0 & 0 \\ \varphi_{3,1} & \varphi_{3,2} & \varphi_{3,3} & 0 & 0 & 0 & 0 \\ \varphi_{4,1} & \varphi_{4,2} & \varphi_{4,3} & \varphi_{4,4} & \varphi_{4,5} & \varphi_{4,6} & \varphi_{4,7} \\ \varphi_{5,1} & \varphi_{5,2} & \varphi_{5,3} & \varphi_{5,4} & \varphi_{5,5} & \varphi_{5,6} & \varphi_{5,7} \\ \varphi_{6,1} & \varphi_{6,2} & \varphi_{6,3} & \varphi_{6,4} & \varphi_{6,5} & \varphi_{6,6} & \varphi_{6,7} \\ \varphi_{7,1} & \varphi_{7,2} & \varphi_{7,3} & \varphi_{7,4} & \varphi_{7,5} & \varphi_{7,6} & \varphi_{7,7} \end{bmatrix} \quad (7)$$

The impulse response functions from this SVAR can be further used to assess the MLC. Let $\frac{dy_i(h)}{d\varepsilon^s(0)} = \theta_i^s(h)$ be the impulse response function of variable i at instant $h \geq 0$ due to a shock ε^s at time $h = 0$. Partial derivatives can be obtained using ratios of the impulse response functions. In particular, $\frac{dtb^s(0)}{dtot^s(0)} = \frac{\theta_{tb}^s(0)}{\theta_{tot}^s(0)}$ since the deviation from pre-shock level $t = 0$ coincides with the notion of a derivative: $\frac{dtb^s(0)}{dtot^s(0)} = \frac{tb^s(0) - \underline{tb}}{tot^s(0) - \underline{tot}} = \frac{\theta_{tb}^s(0)}{\theta_{tot}^s(0)}$, where bars indicate pre-shock level. Since the variables are all expressed in logarithm, the derivative also represents elasticity. Analogously, an approximate measure of elasticity based on the accumulated response from $t = 0$ to any time $t = T$ can be estimated according to $\frac{\Delta tb^s(T)}{\Delta tot^s(T)} = \frac{\sum_{t=0}^{T}\left(tb^s(t) - \underline{tb}\right)}{\sum_{t=0}^{T}\left(tot^s(t) - \underline{tot}\right)} = \frac{\sum_{t=0}^{T}\theta_{tb}^s(t)}{\sum_{t=0}^{T}\theta_{tot}^s(t)}$, where we adopt Δ to distinguish from the instantaneous variation represented by d.

We verify the presence of the MLC by directly evaluating $\frac{\Delta tb^s(0)}{\Delta tot^s(0)}$ and also by separately computing *TOT* elasticity of the volume exported (η_x) and imported (η_m), providing an additional way for checking whether $(\eta_x + \eta_m - 1) > 0$ or not. Analyzing the individual elasticities also allows a better comprehension of the channels responsible for determining the *TB* at the business cycle frequency.

2.3. Data

We work with quarterly data ranging from the 1^{st} semester of 1999 to the 4^{th} of 2019. The beginning of our sample coincides with the adoption of a free-floating regime for the nominal exchange rate together with an inflation targeting for the monetary policy, which replaced the tight crawling peg regime for the nominal exchange rate that lasted in Brazil from the beginning of 1995 to the first week of 1999. Since monetary and fiscal policies vary according to the exchange rate regime, policy responses to global shocks may as well be different, also implying in different reactions by the economic agents. Starting our sample in 1999 places us in a more homogenous environment regarding monetary and exchange rate policies[13].

wgdp is proxied by the log of the sum of the GDP of the countries belonging to the G-20, except Brazil. The subtraction of the Brazilian GDP aims at avoiding any issue related to endogeneity, guaranteeing that shocks to our measure of world activity is orthogonal to the Brazilian[14]. The GDP series are referred in real US dollars of 2015 and were collected in the

[13] Filho and Ferreira (2016) evaluate the presence of a J-curve in Brazil for industrial products of different technological intensity using VECM with annual data ranging from 1996 to 2012. In addition to the very small sample and time horizon, their approach seems inappropriate due to the mixing of periods with different exchange rate and monetary policies.

[14] We consider the subtraction of the Brazilian GDP from the G-20 GDP a conservative measure. Given its size relative to the GDP of the G-20, it is unlikely that oscillations in the GDP of Brazil would contaminate and produce relevant variation in the G-20 GDP.

OECD website already seasonally adjusted. *wp* is proxied by the logarithm of the *all commodities price index* of the IMF, which aggregates prices referred in US dollar of most commodities exchanged worldwide, including energy and non-energy.

The Brazilian national account variables are the seasonally adjusted quarterly series computed and released by the Brazilian Institute of Geography and Statistics (IBGE). We work with the logarithm of GDP (y), household consumption (c), and gross capital formation (i).

All variables associated to the Brazilian commerce with the rest of the world are from FUNCEX (Fundação Centro de Estudos em Comércio Exterior) available in a monthly frequency, which we quarterly aggregate before the seasonal adjustment[15]. FUNCEX estimates the quantum exported (X) and imported (M) using the kilograms exported and imported. The price indexes (P^X and P^M) are obtained after dividing export and import (in US$) by their respective quantum. Following Backus, Kehoe and Kydland (1994) and Senhadji (1998), we proxy *tot* using the log of the ratio between P^M and P^X: $tot = log(P^M/P^X)$. *tb* is the log of the ratio between the total value exported and imported: $tb = log\left(\frac{P^X \times X}{P^M \times M}\right) = x - m - tot$, where $x = log(X)$ and $m = log(M)$. The composition of what is in fuel, capital and consumption goods follows the CGCE/SECEX (Classification by Major Economic Categories[16]) and can be assessed in Table 7 in the appendix.

2.4. VECM versus nonstationary VAR in level

As emphasized in the Introduction, almost every empirical work on MLC and J-curve relies on VECM instead of a VAR with nonstationary variables in level, the latter being our choice. In the Brazilian literature, only Rocha, Magalhães e Brilhante (2024) follow our approach. Several reasons justify our choice. First, our strategy dialogues directly to the general equilibrium models developed Backus, Kehoe and Kydland (1994) and Senhadji (1998) and their impulse response functions are directed to variables in level, although in a stationary environment. Our procedure would certainly be

[15] We seasonally adjusted using X13-ARIMA SEATS, which is the same methodology employed by IBGE. Details are available at <ftp://ftp.ibge.gov.br/Contas_Nacionais/Contas_Nacionais_Trimestrais/ Ajuste_Sazonal/X13_NasContasTrimestrais.pdf>.

[16] In Portuguese, CGCE stands for Classificação por Grandes Categorias Econômicas.

inappropriate if it was problematic from the econometric perspective, but this is not the case. Actually, Sims (1980) and Sims, Stock and Watson (1990), among others, recommend against differencing even if the variables contain a unit root, since the goal of a VAR analysis is to determine the interrelationships among the variables, not the parameter estimates. According to them, the practice of differencing *throws away* information concerning the co-movements in the data (such as the possibility of cointegrating relationships). As exposed by Enders (2008), there is neither a need to detrend the data, since in a VAR a trending variable is well approximated by a unit root plus drift. Even if a researcher is interested on the specific values of estimated coefficients of a VAR with integrated variables, standard least square and maximum likelihood estimators are consistent and asymptotically normal under general conditions and can be used as in stationary cases (see Park and Phillips (1988, 1989), Sims, Stock and Watson (1990), Lütkepohl (2005, Chapter 7), Kilian and Lütkepohl (2017)).

Bayesian estimation methods applied to nonstationary VARs has similar properties, with the advantage of producing impulse responses with smaller standard deviation, since the priors help disciplining the range of the posterior distribution. In what matters for our work, we followed the methodology developed by Zha (1999), which is suitable for modelling a nonstationary global VAR structure for a small open economy since it more properly deals with the block recursion structure. The work of Zha builds on the development of Sims and Zha (1998) on Bayesian estimation and inference of VAR in levels containing non-stationary data.

Estimating VAR models using nonstationary variables in level seems to be the dominant approach in modern applied macroeconomics (for instance, Bernanke *et al.* (1997), Cushman and Zha (1997), Leeper (1997), Kim (2001), Christiano, Eichenbaum and Evans (2005), Antolín-Díaz and Rúbio-Ramirez (2018), Miranda-Agrippino and Ricco (2021), to cite a few). It is thus surprising the still dominant use of differencing and VECM in the MLC and J-curve literature. On this regard, our paper joins Rocha, Magalhães and Brilhante (2024) and contribute to the MLC and J-curve literature by showing how to evaluate their presence based on the widespread approach of estimating VARs with nonstationary variables in level.

3. Estimation and Results

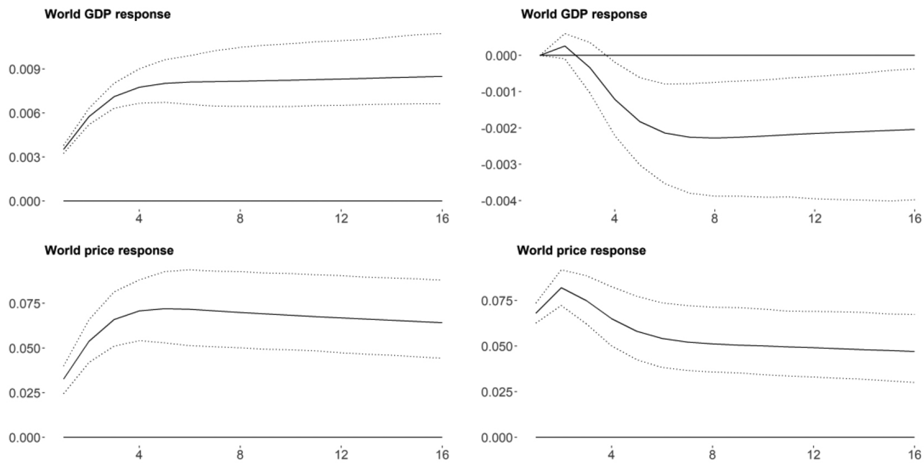
The models are estimated using Bayesian techniques developed by Sims and Zha (1998) and Zha (1999) to help discipline the confidence intervals of the impulse response functions (IRFs). All models use 4 lags ($L = 4$), which is standard for quarterly data. We report median impulse response functions (IRFs) and the 68% confidence interval after 10,000 replications of the Monte Carlo procedure used in the estimation. Since ADF unit root tests failed to reject the null for almost all variables (Table 8 in the appendix), we followed the priors suggested by Sims and Zha (1998) and Zha (1999) that are designed for systems containing non-stationary variables. In the specific case of the VAR for fuel, which contains at least three variables that the tests rejected the null (terms of trade, quantum exported and imported), we modified one of the original priors, placing less weight on the unit root priors for all variables in the system[17]. Details regarding the priors are in the appendix. Regarding the quality of the adjustments, tables 9 to 13 in the appendix show that two different tests fail to reject the null hypothesis of no 1st and 4th order autocorrelation applied to the residuals of every variable in each system, indicating cointegration of the variables and that $L = 4$ is a reasonable lag choice.

3.1. Global shocks and global responses

We start presenting IRFs at the world level to verify if our strategy identifies the shocks suitable to be interpreted as supply and demand. Following Blanchard (1989), Bekaert, Engstrom and Ermolov (2021), Ermolov (2022), and according to standard textbook analysis, a positive (good) world demand shock should increase global economic activity and the commodity price index, while a negative (adverse) supply shock should provoke a fall in global economic activity and a fall in the price index.

The variables and identification strategy are exactly the same in the exercises to analyze the J-curve and the MLC, as one can certify by the first two rows of matrices 3 and 6. And since we estimate both VARs with the same lag order ($L = 4$), they generate the same IRFs for the *wgdp*, *wp*, and *tot*, so one single analysis for the world economy serves both VARs.

[17] We thank one of the referees for warning about this cautious consideration of the priors according to the stationary status of each variable.



(a) Shock: $\varepsilon^{wd} > 0$ **(b)** Shock: $\varepsilon^{ws} > 0$

Figure 1 – Impulse response functions of international variables to global shocks

Note: Dotted lines correspond capture the 68% probability interval. Shocks are identified according to the pairs expressed by equations 3 and 4 or 6 and 7. Source: Prepared by the authors.

Figure 1 presents the IRFs following an unexpected rise in ε^{wd} and ε^{ws} [18]. The positive innovation ε^{wd} provokes a rise in *wgdp* and *wp* (column ***a***), reactions consistent with a positive demand shock. The heighten in ε^{ws} causes *wgdp* to fall and *wp* to elevate (column ***b***), expected movements following an adverse supply shock. We show in the appendix A2 that using an alternative ordering results in IRFs not consistent with what one would expect from supply and demand shocks: while responses to ε^{wd} do not change, a positive perturbation in ε^{ws} augments *wp* and *wgdp*, which is not consistent with a leftward shift in the supply curve.

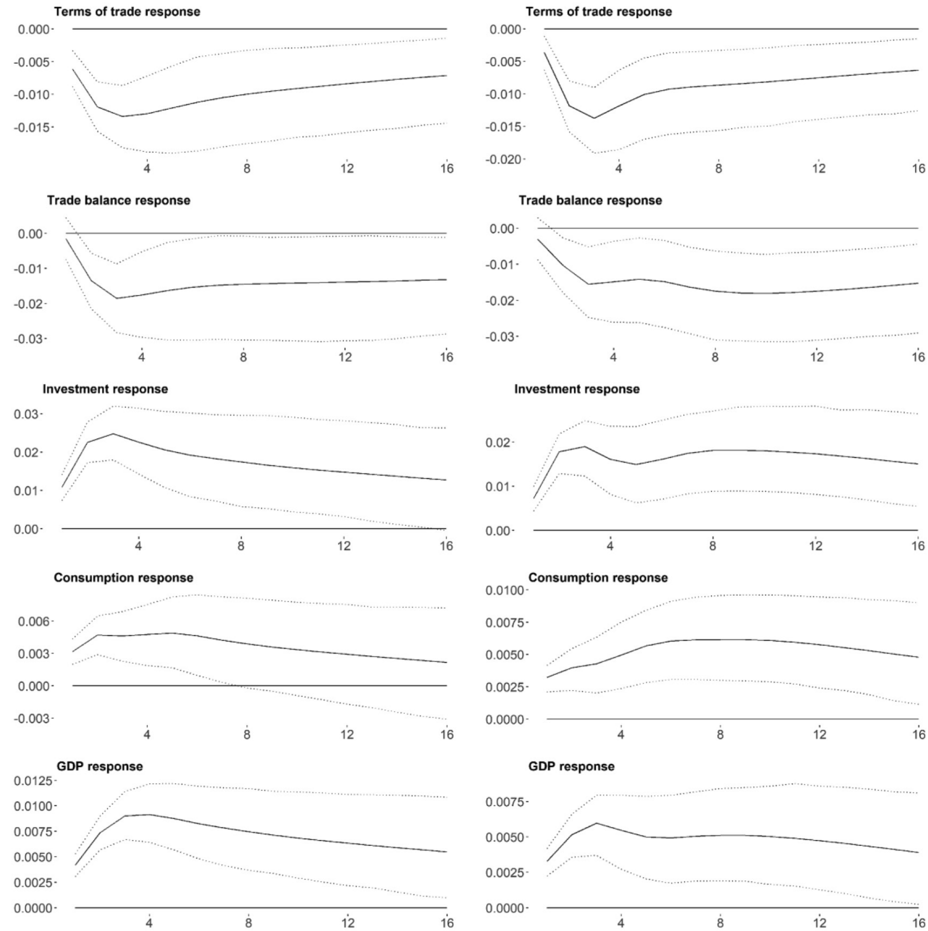
3.2. Global shocks and domestic responses

3.2.1. Is there a J-curve?

The model represented by equations (matrices) 3 and 4 is used to evaluate the occurrence of the J-curve in a general equilibrium perspective where global shocks act as the drivers. The results are in Figure 2. Column ***a***

[18] Numerically, wd=0.35% and ws=6.8%.

displays the IRFs of domestic variables following an increase in ε^{wd}, while column ***b*** does the same for an increment in ε^{wd}.



(a) Shock: $\varepsilon^{wd} > 0$ **(b)** Shock: $\varepsilon^{ws} > 0$

Figure 2 - Impulse response functions of domestic variables to global shocks using model 1

Note: Dotted lines correspond capture the 68% probability interval. Shocks are identified according to the pairs expressed by equations 3 and 4. Source: Prepared by the authors.

The domestic variables react very similar to both innovations. *tot* improves indicating that the p^x increases more than p^m[19]. The appreciation of *tot* already occurs on impact, indicating a fast pass through of world movements to the price of goods Brazil trades internationally. Given the response of *tot*, the presence of a J-curve would imply an initial improvement in *tb* followed by a later deterioration. Our IRFs show, however, a worsening of *tb* already on impact followed by a further deterioration until approximately 1 year after the shock, after which it starts to improve slowly, but persistently remaining below pre-shock path. This entire dynamic implies the absence of a J-curve.

The adjustment of the macroeconomic aggregates that justifies the dynamics of *tb* are in line with those reported by Backus, Kehoe and Kydland (1994) and Senhadji (1998)[20]. The positive global demand shock increases *gdp*, *c*, and *i*. As it is standard in the business cycles literature, the reaction of the last is more intense since households tend to smooth consumption. On impact, *i* increases around 1% and *c* by 0.3%, with their relative difference augmenting over the adjustment. The rise in *gdp* lies between those registered by *c* and *i*, being closer to the first as it corresponds to approximately 65% of the GDP against 18% of the investment. *tb* falls because the domestic absorption expands more than *gdp*.

The response direction of the macroeconomic aggregates is also similar of what is reported by Schmitt-Grohé and Uribe (2018), except for the trade balance, which they find to improve after their positive *terms of trade shocks*, implying in an increase in domestic absorption smaller than that of the *GDP*. While it is hard to step in much further to understand the parts of their model responsible for these differences, the determination of TOT substantially opposes our approaches and may be the source of the differences, as they do not link the oscillations in *TOT* to economic meaningful shocks at the world level.

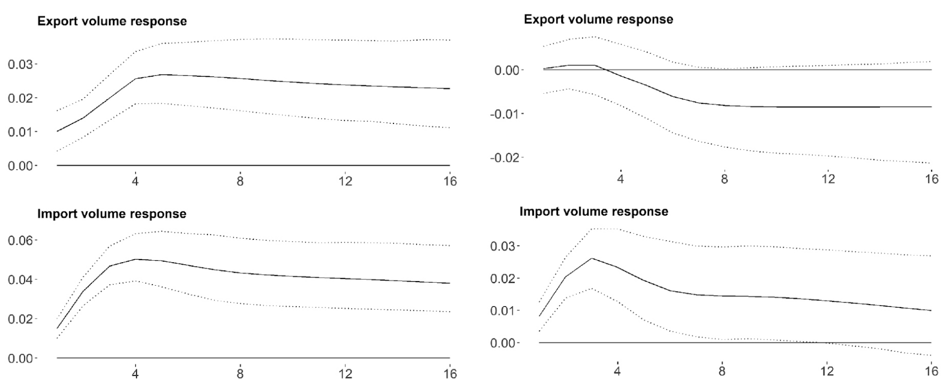
[19] Although an appreciation of the terms of trade not necessarily coincides with that of the real exchange rate, they seem to move in the same direction for the shocks we consider. For instance, Schmitt-Grohé and Uribe (2018) simulate impulse response functions to a "terms of trade shock" using SVAR and DSGE for 38 emerging economies. The median response shows appreciation of the real exchange rate after the improvement in the terms of trade. For the specific case of the Brazilian economy, Ferreira and Valério (2022) reports appreciation of the nominal exchange rate following a positive world demand shock and an adverse world supply shock.

[20] Senhadji (1998) does not analyze the impact of a global supply shock, which limits the comparison.

3.2.2. The Marshall-Lerner condition

We inspect the MLC in the presence of global shocks by considering the SVAR structure represented by matrices (equations) 6 and 7. The only difference between this VAR and the one just analyzed is the replacement of c and i by the index of the volume exported (x) and imported (m), which combined with a similar identification strategy maintains unaltered the IRFs for tot, tb and gdp when compared to the previous VAR[21].

The IRFs are in figure 3, with column a showing the reactions to $\varepsilon^{wd} > 0$ and column b to $\varepsilon^{wd} > 0$. m increases under both shocks, which is consistent with a higher gdp and a more appreciated tot. x responds differently accordingly to the shock: it falls in the presence of $\varepsilon^{ws} > 0$, which is consistent with a more appreciated tot and a smaller $wgdp$; but expands after $\varepsilon^{wd} > 0$, which is consistent with a more intense global activity but not to a more appreciated tot, at least according to standard models that derive the MLC. Despite this opposite response of x, the intense augment in the m reduces tb, guaranteeing the validity of the MLC.



(a) Shock: $\varepsilon^{wd} > 0$ **(b)** Shock: $\varepsilon^{ws} > 0$

Figure 3 – Impulse response functions of the volume exported and imported using model 2

Note: Dotted lines correspond capture the 68% probability interval. Shocks are identified according to the pairs expressed by equations 6 and 7. Source: Prepared by the authors.

[21] Since the responses of this VAR is the same as those reported in the figure used to analyze the J-Curve, we only present the responses of the m and x to the identified shocks. The IRFs of the other variables are available in the working paper version of this work.

Additional information about the size of these reactions is provided in Table 3, where box A focuses on the results following ε^{wd} and box B is reserved to ε^{ws}. Columns 2 to 5 show, respectively, the percentage variation in tot, x, m, and tb. Columns 6 and 7 show *tot* elasticity of the volume exported $(\eta_x = \frac{\Delta x}{\Delta tot})$ and imported $(\eta_m = \frac{\Delta m}{\Delta tot})$, in this order. According to standard models, we should expect $\eta_x > 0$ and $\eta_m < 0$. The last two columns evaluate the presence of the MLC using two measures. The first directly evaluates the sign of the *tot* elasticity of the trade balance $(\frac{\Delta tb}{\Delta tot})$, while the last column evaluates the sign of $\eta_x + \eta_m - 1$[22].

Variations along the 1st year and the cumulative response up to the 3rd indicate the presence of the MLC regardless of the international shock and the measure $(\frac{\Delta tb}{\Delta tot}$ or $\eta_x + \eta_m - 1)$ since they are all positive. Despite the validity of the MLC, the *tot* elasticity of x (η_x) is only positive when exposed to ε^{ws} and after a long period from the shock. These results strengthen the argument raised in the analysis of the IRFs, indicating that income effects play a more significant role in driving trade than substitution effects resulting from changes in relative prices.

Finally, the increase in x following $\varepsilon^{wd} > 0$ and its reduction when exposed to $\varepsilon^{ws} > 0$ strength the impression that they are capturing, respectively, a positive world demand shock and an adverse world supply shock.

Table 3 - ***Aggregate*** **trade response: percentage change and elasticities**

Time interval	Δtot	Δx	Δm	Δtb	η_x	η_m	$\frac{\Delta tb}{\Delta tot}$	$\eta_x - \eta_m - 1$
Box A. Shock: $\varepsilon^{wd} > 0$								
1st year	-4.4%	7.0%	14.6%	-4.4%	-1.6	-3.3	1.0	0.7
cumulative to the 3rd year	-12.4%	27.3%	49.6%	-14.2%	-2.2	-4.0	1.1	0.8
Box B. Shock: 1st year	-4.1%	0.1%	7.8%	-5.2%	-0.02	-1.9	1.3	0.9
cumulative to the 3rd year	-11.0%	-5.8%	19.8%	-18.7%	0.5	-1.8	1.7	1.3

Notes: Δ refers to percentage variation against the pre-shock level. $\eta_x = \frac{\Delta x}{\Delta tot}$ and $\eta_m = \frac{\Delta m}{\Delta tot}$. The Marshall-Lerner is satisfied if the values reported in last two columns are positive.

[22] Note that this expression is the same as $\eta_x + \eta_m - 1$ presented in the previous section and normally used to evaluate MLC, the difference occurring because we are using the sign of η_m as it is estimated, which is negative and implies $-\eta_m > 0$. The models that derive the MLC assume $\eta_m < 0$ (which is what we obtain), but this negative elasticity is preceded by a negative sign in the equation determining m.

Our results indicating the validity of the MLC independent of the shock we identify match the findings of Rocha, Magalhães, and Brilhante (2024), who also estimate a BVAR with variables in level but evaluate response to a *real exchange rate shock*. Regarding the presence of a J-curve, our results differ as they observe the pattern in their estimates. We believe that two reasons may be behind the differences: i) modelling the VAR considering Brazil as a small open economy, as we do, which asks for a bloc recursion structure specifically designed for internationally determined variables; and ii) impulse response analysis to economic meaningful shocks, recognizing that an expressive part of the variation in trade volume and prices is due to shocks happening at the world level. We do not analyze the source of the differences, but it is certainly an exercise worthwhile pursuing.

4. Disaggregate Responses

We also evaluate the responses of trade variables related to capital goods, consumption goods, and fuel. The interest in evaluating the first two is related to concerns regarding the Dutch Disease, which, if present, should more probably be encountered in the responses of final non-commodity goods such as capital and consumption goods, which also tend to have longer production chains[23]. The exercise for fuel seems interesting because corresponds to a commodity aggregate that Brazil both imports and exports. These analyses rely on a similar econometric strategy used to evaluate the MLC condition (equations 6 and 7), but now using the volume exported and imported and *tot* specific to each good.

4.1. Capital goods

Figure 4 shows the IRFs of the variables associated to trade in capital goods. When exposed to $\varepsilon^{wd} > 0$, *tot* improves on impact, the same direction observed for the aggregate *tot* but in a less intense magnitude. *m* and *x* also follow the aggregate pattern and increase. The first reacts consistently with a higher *gdp* and a more appreciated *tot*, while the second responds consistently to higher *wgdp* but not to the improvement in *tot*.

[23] Note that our focus on evaluating capital and consumption goods is not related to their importance in the Brazilian trade balance, which is why we do not pursue an analysis directed to intermediate goods.

The net effect is an improvement in the *tb* of capital goods, with the world income exerting a more relevant role than the substitution effect provoked by changes in relative prices. The world income effect is so intense that compensates not only the price effect, but also the domestic income effect that also contributes with the elevation of imports of capital.

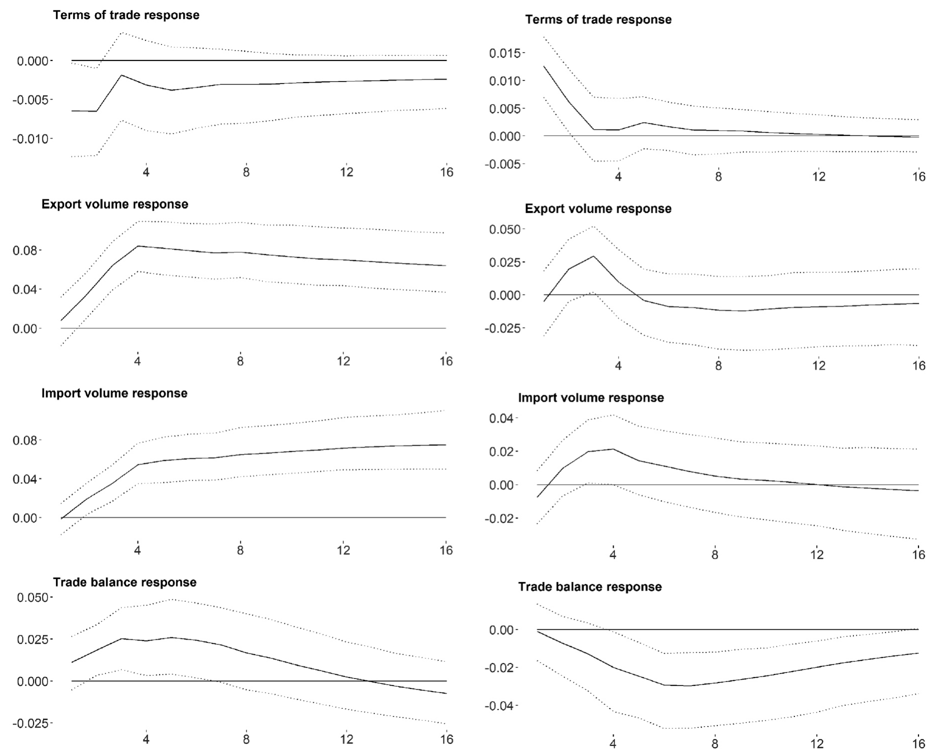
When exposed to $\varepsilon^{ws} > 0$, *tot* of capital goods depreciates on impact and rapidly reverts to the pre-shock trend after 3 quarters. This reaction is opposite to the impact appreciation verified for the aggregate *tot*. *x* surprisingly increases, which probably reflects price stimulus since *wgdp* falls behind the pre-shock trend. This increase in *x* does not appear to be associated with a shift from the local to the world market, since *gdp* and *i* increase. *m* also increases, which is consistent with a higher *gdp* and *i*, but not to a more depreciated *tot*. The worsening in the *tb* of capital goods also runs in opposition to the depreciation of its *tot*, indicating the prevalence of domestic income effect over price incentives. The J-curve is not observed after the shocks.

Table 4 – ***Capital Goods*** **trade response: percentage change and elasticities**

Time interval	Δtot	Δx	Δm	Δtb	Ηx	ηm	$\frac{\Delta tb}{\Delta tot}$	$\eta_x - \eta_m - 1$
Box A - shock ε^{wd}								
1st year	-1.8%	19.0%	10.7%	7.8%	-10.6	-5.9	-4.4	-5.7
cumulative to the 3rd Year	-4.3%	79.5%	63.0%	19.8%	-18.7	-14.8	-4.7	-4.9
Box B - shock ε^{ws}								
1st year	2.1%	5.3%	4.3%	-4.1%	2.5	2.1	-1.9	-0.5
cumulative to the 3rd Year	2.9%	-2.3%	39.0%	-24.5%	-0.8	3.0	-8.3	-4.8

Notes: Δ refers to percentage variation against the pre-shock level. $\eta_x = \frac{\Delta x}{\Delta tot}$ and $\eta_m = \frac{\Delta m}{\Delta tot}$. The Marshall-Lerner is satisfied if the values reported in last two columns are positive.

Table 4 shows that $\frac{\Delta tb}{\Delta tot}$ and $\eta_x - \eta_m - 1$ and are negative regardless of the shock and time interval, indicating that MLC is not verified for capital goods. Focusing on each elasticity, η_x is negative following ε^{ws}, which is contrary to the expected sign according to standard theory and further revealing the important role played by the world income. The sign of η_m is in accordance with standard theory behind MLC under ε^{wd}, but not in the presence of ε^{ws}, being negative in the first case and positive in the second. This last result indicates the prevalence of domestic income effect that acts in the direction of elevating the absorption regardless the price incentive.



(a) Shock: $\varepsilon^{wd} > 0$ **(b)** Shock: $\varepsilon^{ws} > 0$

Figure 4 - Capital goods: impulse response functions of domestic variables to global shocks - MLC condition

Note: Dotted lines correspond capture the 68% probability interval. Source: Prepared by the authors.

4.2. Consumption goods

Figure 5 shows that the IRFs associated with consumption goods behave very similar to the aggregate measures of trade. Following $\varepsilon^{wd} > 0$, there is an improvement in *tot*, *x*, and *m*, but a worsening in *tb*, although not significantly. On impact, however, *tb* presents a non-significant increase, but it is very small to allow for any consideration regarding the presence of a J-curve. Again, the import response is coherent with higher *gdp* and appreciated *tot*, while the augment in the *x* is consistent with higher *wgdp*, but inconsistent with the price incentive.

The innovation $\varepsilon^{ws} > 0$ also improves tot, which reverts to pre-shock levels 4 quarters later. This adjustment is faster and less persistent than reported for the aggregate tot. Despite the improvement in tot and the reduction in $wgdp$, the volume sold abroad remains stable. Pricing to market, low price, and income elasticity of the world demand for Brazilian consumption goods following the shock may all explain that pattern. On the other hand, the increase in m is consistent with the improvement in tot and with higher gdp. The net result is a decrease in the tb of consumption goods that persistently remains below pre-shock levels. The J-curve is not observed[24].

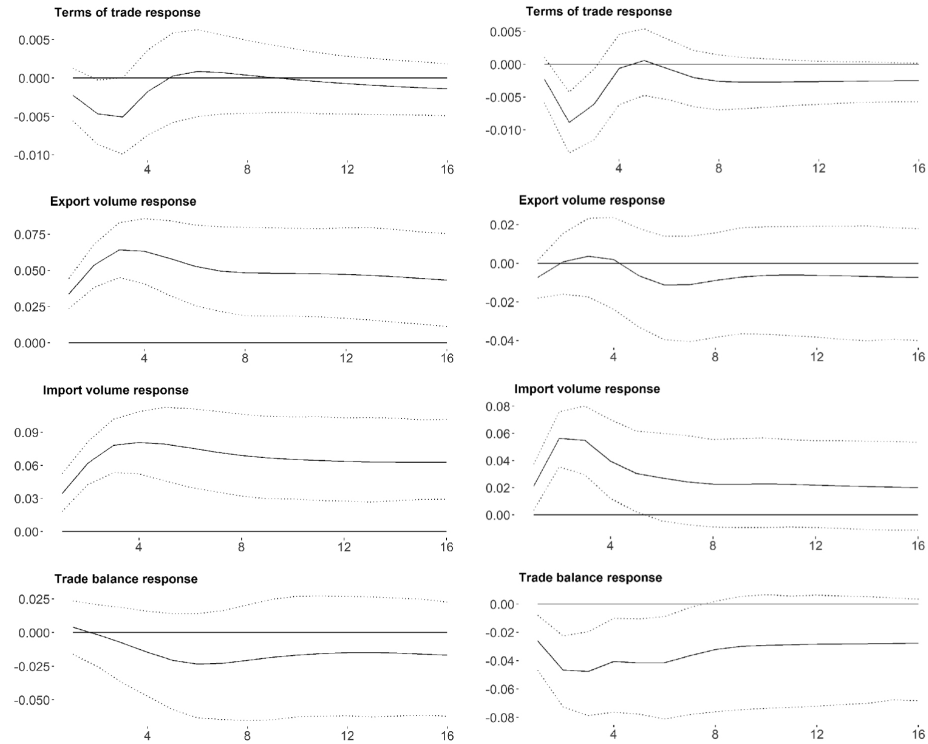
Table 5 shows that $\frac{\Delta tb}{\Delta tot}$ and $\eta_x - \eta_m - 1$ are positive regardless the shock, validating the MLC. Similarly, η_m is always negative, also conforming to the hypotheses behind the MLC, the same happening with η_x that is positive in the presence of ε^{ws}. Despite of behaving as expected when strictly considering the relation between tb and tot, the reactions are also consistent with the relation between tb and income (domestic and international), not being possible to say which is playing the most important role. This changes when observing that $\eta_x < 0$ in the presence of ε^{wd}, which opposes the expectation under the MLC and shows the prominent role played by the world income.

Table 5 – ***Consumption Goods*** **trade response: percentage change and elasticities**

Time interval	Δtot	Δx	Δm	Δtb	ηx	ηm	$\frac{\Delta tb}{\Delta tot}$	$\eta_x - \eta_m - 1$
Box A - shock ε^{wd}								
1^{st} year	-1.4%	21.5%	25.5%	-2.0%	-15.7	-18.6	1.5	1.9
cumulative to the 3^{rd} Year	-1.3%	61.5%	81.2%	-17.3%	-47.9	-63.3	13.5	14.3
Box B - shock ε^{ws}								
1^{st} year	-1.8%	-0.1%	17.2%	-16.1%	0.05	-9.7	9.0	8.7
cumulative to the 3^{rd} Year	-3.3%	-6.4%	36.7%	-42.8%	1.9	-11.0	12.8	11.9

Notes: Δ refers to percentage variation against the pre-shock level. $\eta_x = \frac{\Delta x}{\Delta tot}$ and $\eta_m = \frac{\Delta m}{\Delta tot}$. The Marshall--Lerner is satisfied if the values reported in last two columns are positive.

[24] Our findings indicating the validity of the MLC for consumer goods is also observed by Arruda, Brito, and Castelar (2022) despite of their use of a different approach. But we find different results for capital goods since they observe the MLC. In the case of the J-curve, they claim to have observed in some situations, but the absence of confidence intervals for their estimates makes it hard to assess whether the results are statistically different from zero.



(a) Shock: $\varepsilon^{wd} > 0$ **(b)** Shock: $\varepsilon^{ws} > 0$

Figure 5 – Consumption goods: response of domestic variables to global shocks - MLC condition

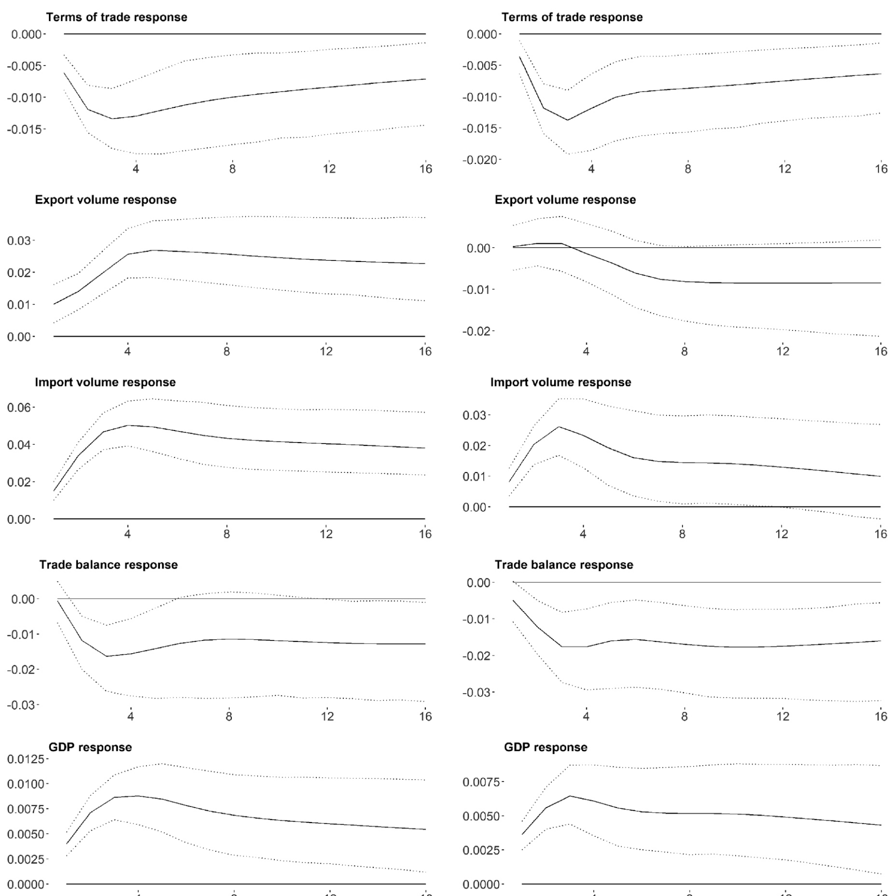
Note: Dotted lines correspond capture the 68% probability interval. Source: Prepared by the authors.

4.3. Fuel

Analyzing the trade in fuel is particularly interesting because Brazil imports and exports the commodity. As table 1 shows, exports of fuel corresponded to 13.3% of the total exports in 2019, while imports represented 11.7% of the aggregate imports. Its trade balance of US$9.4 billion corresponded to 19.6% of the country’s total in 2019. Trade occurs because most Brazilian refineries use oil with a density that is different from a large share of what is extracted domestically.

Figure 6 shows the fuel *tot* improving on impact after both shocks. This appreciation continues until 3 quarters after the shock, with the start of a reversion towards pre-shock level occurring after 1 year, being though incomplete even after 4 years after the shock, signaling a large persistence regardless the shock.

In the presence of $\varepsilon^{wd} > 0$, x and m increase remain above pre-shock trend in a persistent manner during the 4 years we simulate. The behavior of x is consistent with a higher $wgdp$, but not with a more appreciated tot, while m response is consistent with higher gdp and a more appreciated tot. tb falls below pre-shock trend, validating the MLC despite of x responding contrary to the hypothesis used to develop the MLC. The dynamic response of tb does not show a J-curve pattern.



(a) Shock: $\varepsilon^{wd} > 0$ **(b)** Shock: $\varepsilon^{ws} > 0$

Figure 6 - Fuel: impulse response functions of domestic variables to global shocks - MLC condition

Note: Dotted lines correspond capture the 68% probability interval. Source: Prepared by the authors.

When exposed to $\varepsilon^{ws} > 0$, x does not react on impact but falls later, persistently remaining 1% below pre-shock trend. The pattern is consistent with smaller $\boldsymbol{wgdp}$ and an appreciated tot. $\boldsymbol{m}$ increases, which can be associated with higher $\boldsymbol{gdp}$ and a more appreciated $\boldsymbol{tot}$. $\boldsymbol{tb}$ falls below pre-shock trend validating the MLC. A J-curve pattern is not verified.[25]

Table 6 – *Fuel* trade response: percentage change and elasticities.

Time interval	Δtot	Δx	Δm	Δtb	ηx	ηm	$\frac{\Delta tb}{\Delta tot}$	$\eta_x - \eta_m - 1$
Box A - shock ε^{wd}								
1^{st} year	-4.4%	7.0%	14.6%	-4.4%	-1.6	-3.3	1.0	0.7
cumulative to the 3^{rd} year	-12.4%	27.3%	49.6%	-14.2%	-2.2	-4.0	1.1	0.8
Box B - shock ε^{ws}								
1^{st} year	-4.1%	0.1%	7.8%	-5.2%	0.0	-1.9	1.3	0.9
cumulative to the 3^{rd} year	-11.0%	-5.8%	19.8%	-18.8%	0.5	-1.8	1.7	1.3

Notes: Δ refers to percentage variation against the pre-shock level. $\eta_x = \frac{\Delta x}{\Delta tot}$ and $\eta_m = \frac{\Delta m}{\Delta tot}$. The Marshall-Lerner is satisfied if the values reported in last two columns are positive.

Table 6 shows that $\frac{\Delta tb}{\Delta tot}$ and $\eta_x - \eta_m - 1$ are positive for the accumulated response up to the 1^{st} and 3^{rd} years, validating the MLC in both time intervals. As already emphasized, the export elasticity is negative under ε^{wd}, contradicting the hypothesis of the models deriving the MLC. Once again, the income effect reveals to be more important than price effects, and the large (negative) elasticity of the volume imported guarantees $\eta_x - \eta_m - 1 > 0$.

Since we estimate the VAR for fuel placing less weight on the sum-of-coefficients prior (see section 3 and appendix A2) because of the rejection of the null of unit root (Table 8 in the appendix) for *fuel variables* (tot, x, m, and tb), we decided to show the reaction of gdp in the last row of Figure 6 to compare with its displayed in Figure 2, when the system was estimated with $\mu_5 = 1$, the value originally proposed by Sims and Zha (1998), Zha (1999), and Cushman and Zha (1997). The comparison reveals similar reactions regardless the values for μ_5, indicating that placing less weight on this specific prior to accommodate for the presence of stationary variables does not affect the responses of the non-stationary of variables present

[25] Our results validating the MLC for trade in fuel regardless the shock (and also regardless of estimating the VAR with 5=1 or 0.5) opposes the findings of Arruda, Brito and Castelar (2022).

in the VAR[26]. On the other hand, the responses of the stationary fuel variables are more influenced by μ_5, as can be assessed by comparing responses in Figures 6 and 8 in the appendix. This robustness check suggests as reasonable our choice to modify the value for μ_5 when estimating the VAR for fuel.

The comparison of the IRFs of the VAR for fuel with the other systems also reveal that the persistent responses of some variables (that do not revert to pre-shock trend - Figures 1, 2, 3, 4, and 5) is not restricted to non-stationary variables in the systems, suggesting a high persistence in the adjustment of the variables of our systems. A bad decision regarding the number of lags used in the VARs could be a potential explanation for this high persistence. If this was the case, one would expect to find significant autocorrelation in the residuals, but this is not what we verify since we fail to reject the null of no significant 1^{st} and 4^{th} order autocorrelation in the residuals, as shown in tables 9 to 13 in the appendix.

5. Discussion

The relation between *TOT* and *TB* is not consensual. The Harberger-Laursen-Metzler and Obstfeld-Razin-Svensson effects predict an augment in *TB* following an improvement in the *TOT*. Indeed, Schmitt-Grohé and Uribe (2018) estimate that an appreciation in *TOT* ameliorates the *TB* in 38 out of the 51 countries researched after exposing them to an exogenous shock directly applied to an $AR(1)$ process followed by each country's *TOT*. We encounter opposite results: over the business cycle, an improvement in the Brazilian *TOT* is associated with a worsening in *TB*, which is the expected relation if the MLC is satisfied. An important difference between our methods is that we treat *TOT* as an endogenous variable governed by the state of the world economy, being closer in nature to the DSGE models of Backus, Kehoe and Kydland (1994) and Senhadji (1998) who also verify a deterioration in *TB* in the presence of an improvement in *TOT* provoked by a positive global demand shock.

[26] We also compared the impulse responses of the *wgdp* and *wp* using 5=1 (originally proposed by the mentioned papers) and 5=0.5, and observed no difference. We do not report their responses due to space limitations, but we can provide this additional figure under request.

Another important message from our results is that the conformity with MLC does not imply that all derivatives that form the condition need to behave as hypothesized by the relation. This again can be perceived after allowing for general equilibrium adjustments. For instance, a positive world demand shock improves the Brazilian *TOT* and elevates the volume imported and exported. This (conditional) correlation between export and *TOT* is opposite to what is expected according to most international economics models, including those deriving the MLC. Nevertheless, we still observe the validity of MLC thanks to the income effect: the accentuated rise in the domestic income elevates the imported volume in a proportion that more than compensates for the increase in exports.

It is important to emphasize that the pattern we describe for Brazil cannot be automatically extended to other countries without further empirical analysis. For instance, Charnavoki and Dolado (2012) conduct a similar exercise for Canada, a rich small open economy that also possesses a very strong commodity sector. According to their estimates, a positive innovation to the global demand generates several common results as ours: *TOT*, GDP, consumption, and investment ameliorate. But the Canadian *TB* improves (after an initial drop on impact) while the Brazilian worsens.[27] Understanding the causes of these differences deserves future investigation that should address the structural characteristics of each country. Another promising research venue relates to incorporating these characteristics in DSGE models to obtain reaction functions aligned with VAR estimates. For instance, it may a challenge to obtain an augment in local GDP, consumption and investment following an adverse global supply shock.[28]

Our results also allow some consideration regarding the presence of the so-called Dutch disease, an issue constantly debated in the interface of international economics with the growth/development literature. As discussed by Charnavoki and Dolado (2012), the Dutch disease may occur

[27] It is not possible to compare our results of a negative world supply shock with those reported by Charnavoki and Dolado (2012), because they consider two supply shocks: one specific to the commodities market and a broader one. The impulse responses of the international variables show that this broader supply shock is closest to ours, but they do not report the domestic responses following this shock.

[28] Ferreira and Valério (2022) also observe an increase in the GDP of Brazil, Chile, Colombia, and Peru after an adverse global supply shock that increases commodity prices. Chen and Rogoff (2003), Shousha (2016), Drechsel and Tenreyro (2018), Fernández, González and Rodriguez (2018), among others, also encounter a positive relation between commodity prices and the GDP of commodity exporters' countries, even though they do not identify the source of the world innovation responsible for moving the prices.

when a country with a strong commodity sector is hit by a shock that improves its *TOT* and appreciates the *RER*, reducing the competitiveness of the non-commodity tradeable sector and causing its production to fall. We do not evaluate sectoral production, but the disaggregate trade analysis reveals no detrimental effects in the competitiveness of the non-commodity sectors resulting from the improvement of *TOT*. The impulse responses show that when exposed to a positive global demand shock that improves *TOT*, the volume exported of capital and consumption goods increase, opposing the expected drop if the Dutch disease was present. When exposed to a negative global supply shock that also improves the Brazilian *TOT*, the exported volume of capital goods even increases, while those of consumption goods remains mostly constant. To the extent that the world GDP falls and the domestic increases, this positive response in the volume exported of capital goods and the neutral response of consumption goods are hard to reconcile with any symptom of Dutch disease, at least in the business cycle frequency.

6. Concluding Remarks

We evaluate the presence of the J-curve and the Marshall-Lerner condition in Brazil, where commodity exporting plays a very important role in the determination of the trade balance. The analyses rely on the estimation of a global SVAR with characteristics resembling the general equilibrium structure of Backus, Kehoe and Kydland (1994) and Senhadji (1998), which allows the terms of trade to be endogenously determined by world shocks along the business cycle frequency. This distinguishes our paper from most empirical works with similar objective, as they tend to evaluate responses following a *terms of trade shock* instead of considering the drivers behind movements in the terms of trade.

Our impulse response functions reveal the absence of the J-curve after exposing the Brazilian economy to global supply and demand shocks, the same happening when restricting the exercise to trade of fuel, and capital and consumption goods. The relation between the TB and TOT happens as expected according to the Marshall-Lerner condition for aggregate trade, fuel and consumption goods, but not for capital goods. However, the volume exported does not always respond as expected according to the hypotheses behind the MLC and by most international economics models since it improves on several occasions when the TOT appreciates. This apparent puzzle occurs because the volume exported is more responsive to shifts in world GDP than to relative prices. Similar impression applies to the volume imported that seems to be most impacted by movements in domestic income than to relative prices.

Finally, we also observe expansion in the volume exported of capital and consumption goods following an appreciation of TOT, which distances Brazil from concerns about the Dutch disease, at least in the business cycle frequency.

Thinking of future steps, we see room for general equilibrium models for small open economies to incorporate specific characteristics of countries to help explaining different responses to the same international shock. Similarly, including specific forms of heterogeneity in general equilibrium models may provide a better understanding for some results we obtain, like the validity of the MLC for aggregate and consumption goods, but not for capital goods. From the empirical point of view, we see two important routes. One is to apply our methodology to other countries and explore the reasons behind different responses to similar shocks. Another interesting path is to verify if MLC and the J-curve are state dependent[29], which is an exercise that is nowadays facilitated by relying on nonlinear local projection techniques.

[29] Ribeiro, Vasconcelos and Silva (2021) verify that the export of several industrial sectors of Brazil respond in a nonlinear manner to *exchange rate shocks*. Important to emphasize, however, that they do not identify the shocks responsible for driving the exchange rate, which is treated as an exogenous variable, an approach we criticize over this article.

7. Appendix

A1. The composition of the disaggregated trade variables

Table 7 - Tradable goods composition

Capital Goods (BK)
1. Capital goods other than industrial transport equipment
2. Industrial transport equipment
Consumption Goods (BC)
1. Consumer durables goods
1.1 Durable consumer goods except transport equipment
1.2 Passenger cars
1.3 Non-industrial transport equipment
Fuel
1. Basic fuels and lubricants
2. Prepared fuels and lubricants

Source: Methodological Note DEAEX/SECEX/CGET nº 001/2016 - https://balanca.economia.gov.br/balanca/metodologia/Nota_CGCE.pdf

A2. Bayesian priors for the VAR estimation

We estimate the Bayesian VAR using the priors suggested by Sims and Zha (1998), Zha (1999), and Cushman and Zha (1997). The last reference also imposes a block recursive restriction to model Canada as a small open economy. Two unit root priors are combined: the Minnesota prior and the sum-of-coefficients prior. The first imposes that coefficients on the first lag has prior mean of 1. According to Sims and Zha (1998) this is done by creating variables such that for the ith equation a set of $k-1$ dummy observations, indexed by and $j = 1,\dots,m$, and $l = 1,\dots,p$ is inserted in the data sample according to

$$y_i(r,j)\ ;\ r=1,\dots,k-1;\ j=1,\dots,m=\begin{cases}\mu_1\mu_2\,\sigma_r/l^{\mu_4}, & \text{if } r=j, r\le m\\ 0, & \text{otherwise}\end{cases}$$

$$x_i(r,s)\ ;\ r=1,\dots,k-1;\ s=1,\dots,k-1=\begin{cases}\mu_1\mu_2\,\sigma_r/l^{\mu_4}, & \text{if } r=s,\\ 0, & \text{otherwhise}\end{cases}$$

where μ_1, μ_2 and μ_4 are hyperparameters. μ_1 controls the overall tightness of $A(0)$; μ_2 controls the relative tightness of $A(L)$ for $L>0$, and μ_4 controls the tightness on lag decay. We set these hyperparameters at the default values suggested by Sims and Zha (1998), which are 1, 0.5 and 1, respectively.

The sum-of-coefficients prior is used in cases where the variables have a unit root, which is attained by setting to 1 the sum of the lagged coefficients of each dependent variable. In a system of m equations, l lags k and coefficients, it is introduced m observations, indexed by i, according to:

$$y(i,j)\ ;\ i=1,\dots,m;\ j=1,\dots,m=\begin{cases}\mu_5\bar{y}_{0i}, & \text{if } i=j,\\ 0, & \text{otherwise}\end{cases}$$

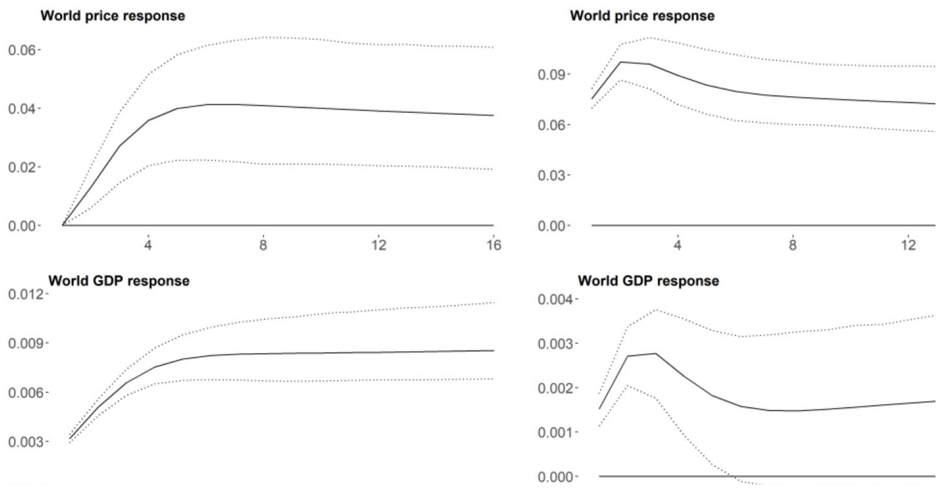
$$x(i,s)\ ;\ i=1,\dots,m;\ s=1,\dots,k=\begin{cases}\mu_5\bar{y}_{0i}, & \text{if } i=j, \text{for all } l,\\ 0, & \text{otherwise}\end{cases}$$

where $\underline{y}_{0i}$ is the average of initial values of the variable i and μ_5 is a hyperparameter that controls the weight of the prior. When $\mu_5\to\infty$, the model can be expressed entirely in terms of differenced data. We follow Sims and Zha (1998), Zha (1999), and Cushman and Zha (1997) and set $\mu_5 = 1$ in every estimate, except for the VAR for fuel, which we set it to 0.5. The reason is that previous analysis (Table 8 in the appendix) indicates the rejection of the unit root hypothesis in most variables related to the fuel (*tot*, *x* and *m*, and even *tb* depending on the model especification.). Important to remember that the VAR for fuel still contains variables (*wgdp*, *wp*, and *gdp*) that most likely have a unit root or a linear trend according to table 8. By setting $\mu_5 = 0.5$ we simply reduce the weight placed on this unit root prior, which does not mean that the estimated coefficients of a non-stationary variable will not correct capture its characterisc[30].

[30] The IRFs of the VAR for fuel using 5=1 and 5=0.5 show similar responses for the non-stationary variables (*wgdp*, *pcom*, and *gdp*). The similarity remains when comparing to the responses of these variables from the other VARs estimated in the paper.

A3. Identification of the world shocks using a different ordering

Figure 7 shows the impulse response functions of the world variables using a different ordering from what we present in section 2.1. Now the world commodity price index comes first in the VAR, with the global GDP coming in second.



(a) Shock: $\varepsilon^{wgdp} > 0$ **(b)** Shock: $\varepsilon^{wp} > 0$

Figure 7 - Impulse response functions of international variables to global shocks: alternative ordering with the price of commodities coming first in the VAR

Note: Impulse responses identified from a different ordering of the international variables from those expressed by the pairs of equations 3 and 4 or 6 and 7. Dotted lines correspond capture the 68% probability interval. Source: prepared by the authors.

The price index increases after a positive shock in the equation of the world GDP, which is the expected reaction following a positive demand shock. However, global GDP also increases after an unexpected shock that elevates the price level. This reaction of wgdp is not consistent with the response from an adverse supply shock, differently from the identification strategy we use throughout the paper.

A4. Unit root ADF tests

Table 8 - t-statistics of ADF tests

Variable	trend + drift	drift	none
$wgdp$	-3.5786**	-0.3534	4.0543
wp	-1.7638	-1.907	0.6636
Tot	-1.8733	-1.7319	-1.3232
Nx	-1.8727	-1.8418	-0.8432
I	-1.2269	-1.5543	0.9707
C	-0.6706	-1.2284	3.0231
gdp	-0.7152	-1.8135	2.5403
X	-2.0391	-2.3037	2.268
M	-2.0585	-1.4111	1.1605
tot_{capital}	-3.0225	-2.2098	-1.2283
x_{capital}	-1.7921	-1.1974	-0.7744
m_{capital}	-1.5392	-1.1241	0.9131
tb_{capital}	-1.7921	-1.1974	-0.7744
tot_{consumer}	-2.4824	-2.1355	-1.7464*
x_{consumer}	-2.3864	-2.4006	-0.1055
m_{consumer}	-1.5408	-1.9155	0.3529
tb_{consumer}	-1.9168	-1.8961	-2.0244**
tot_{fuel}	-4.4463***	-4.1008***	-4.1279***
x_{fuel}	-3.6401**	-3.0224**	1.8846
m_{fuel}	-3.7618**	-3.1381**	0.5712
tb_{fuel}	-2.6084	-2.3823	-3.0415***

Source: Prepared by the authors. Notes: *, **, and *** indicate rejection at 10%, 5%, and 1% critical level, respectively. Columns 2, 3, and 4 display, respectively, the tstat for the ADF unit root test in the presence of "trend+drift", "drift", and no deterministic component. Lag selection for each regression based on BIC.

A5. Tests of autocorrelation in the residuals

Tables 9 to 13 present the first and fourth order autocorrelation for the residuals of each variable in each of the 5 models we estimate. We also present the p-values for testing the null of no autocorrelation according to the traditional Ljung-Box test (LB), and to the Dalla, Giraitis, and Phillips (2022) test (based on their $\tilde{Q}$ statistic), which corrects the LB for its trend to being oversized. Since $wgdp$ and wp form an independent block in each equation, we just report their results for the first system (Table 9). For similar reasons we only report the AR tests for the residuals of *tot* for the first system. Although the domestic GDP enters in every system, in each of them there is a different set of covariates that could alter the estimated residuals in the equation of GDP, justifying its testing in every system.

The results show that the presence of unit roots in the residuals can be very easily discarded, since all first order autocorrelation are very small and the pvalues of both tests are relatively high. The residuals of the aggregate exported volume in the system AML (table 10) is the only exception, since the *pvalues* associated with the first order autocorrelation is below 5%, suggesting the presence of first some autocorrelation in the residual. But the estimated first order autocorrelation still very small, at -0.2877, which is clearly very far from any concerns regarding the presence of a unit root. All tests applied to the 4th order autocorrelation also fail to reject the null, once again indicating a good fitting of the models. Overall, the tests indicate that our use of level variables containing unit root does not pose an econometric problem, since the chosen variables are clearly cointegrated.

Table 9 - Autocorrelation tests in the residuals of each equation of the A_J system

variable	autoc(1)	LB-pval	$\tilde{Q}$-pval	autoc(4)	LB-pval	$\tilde{Q}$-pval
wgdp	0.1603	0.1440	0.4463	0.0214	0.6757	0.9579
Wp	0.0844	0.4418	0.5862	0.0510	0.7486	0.9020
tot	0.0524	0.6329	0.6396	-0.0543	0.9163	0.9113
Tb	-0.0081	0.9411	0.9210	-0.0123	0.9926	0.9791
I	-0.0380	0.7289	0.7058	-0.0627	0.2671	0.2770
C	-0.0579	0.5975	0.6711	-0.0584	0.2939	0.3464
gdp	-0.1144	0.2971	0.3022	-0.0330	0.1257	0.0889

Source: Prepared by the authors. Autoc(1) and autoc(4) refer, respectively, to the 1st and 4th order autocorrelation. LB-pval stands for the p-value according to the Ljung-Box test, and $\tilde{Q}$-pval for the p-value according to the $\tilde{Q}$ statistic of Dalla, Giraitis, and Phillips (2022).

Table 10 - Autocorrelation tests in the residuals of each equation of the A_{ML} system

Variable	autoc(1)	LB-pval	$\tilde{Q}$-pval	autoc(4)	LB-pval	$\tilde{Q}$-pval
X	-0.2877	0.0087	0.0107	0.1603	0.0506	0.0521
M	-0.1274	0.2455	0.2930	-0.0175	0.6906	0.7655
Tb	-0.1151	0.2944	0.3126	-0.0474	0.6366	0.5687
Gdp	-0.0090	0.9344	0.9362	-0.0362	0.8058	0.7972

Source: Prepared by the authors. Autoc(1) and autoc(4) refer, respectively, to the 1st and 4th order autocorrelation. LB-pval stands for the p-value according to the Ljung-Box test, and $\tilde{Q}$-pval for the p-value according to the $\tilde{Q}$ statistic of Dalla, Giraitis, and Phillips (2022).

Table 11 - Autocorrelation tests in the residuals of each equation of the system for capital goods

Variable	autoc(1)	LB-pval	$\tilde{Q}$-pval	autoc(4)	LB-pval	$\tilde{Q}$-pval
tot	-0.0266	0.8085	0.7856	0.0570	0.7864	0.8276
X	-0.0737	0.5020	0.4280	-0.1614	0.5317	0.5947
M	-0.0320	0.7709	0.8214	0.0840	0.8944	0.9202
Tb	0.0230	0.8339	0.8565	0.0893	0.8143	0.8303
Gdp	-0.0164	0.8813	0.8631	0.0081	0.8608	0.8060

Source: Prepared by the authors. Autoc(1) and autoc(4) refer, respectively, to the 1st and 4th order autocorrelation. LB-pval stands for the p-value according to the Ljung-Box test, and $\tilde{Q}$-pval for the p-value according to the $\tilde{Q}$ statistic of Dalla, Giraitis, and Phillips (2022).

Table 12 - Autocorrelation tests in the residuals of each equation of the system for consumption goods

Variable	autoc(1)	LB-pval	$\tilde{Q}$-pval	autoc(4)	LB-pval	$\tilde{Q}$-pval
Tot	-0.0594	0.5908	0.5846	-0.0318	0.8726	0.8644
X	-0.0045	0.9672	0.9699	-0.0426	0.8563	0.9032
M	0.1084	0.3260	0.3868	0.0184	0.4406	0.6062
Tb	0.0714	0.5177	0.5072	0.0596	0.8882	0.8694
Gdp	-0.0328	0.7661	0.7197	-0.0276	0.9449	0.9296

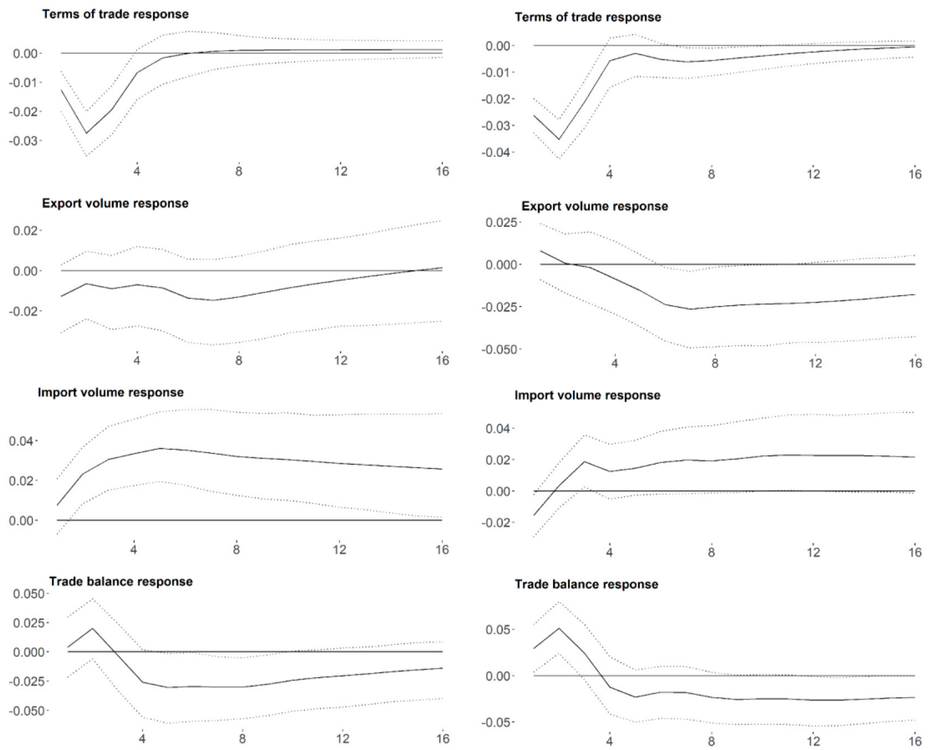
Source: Prepared by the authors. Autoc(1) and autoc(4) refer, respectively, to the 1st and 4th order autocorrelation. LB-pval stands for the p-value according to the Ljung-Box test, and $\tilde{Q}$-pval for the p-value according to the $\tilde{Q}$ statistic of Dalla, Giraitis, and Phillips (2022).

Table 13 - Autocorrelation tests in the residuals of each equation of the system for fuel

Variable	autoc(1)	LB-pval	$\tilde{Q}$-pval	autoc(4)	LB-pval	$\tilde{Q}$-pval
Tot	0.0077	0.9445	0.9440	-0.0017	0.6718	0.6991
X	-0.1907	0.0842	0.0853	0.1638	0.2552	0.2552
M	0.0212	0.8477	0.8710	0.1335	0.5027	0.4566
Tb	0.1195	0.2789	0.2826	-0.1569	0.4488	0.6550
Gdp	-0.1183	0.2840	0.3726	0.031	0.3168	0.3975

Source: Prepared by the authors. Autoc(1) and autoc(4) refer, respectively, to the 1st and 4th order autocorrelation. LB-pval stands for the p-value according to the Ljung-Box test, and $\tilde{Q}$- pval for the p-value according to the $\tilde{Q}$ statistic of Dalla, Giraitis, and Phillips (2022).

A6. Model for fuel using alternative priors: μ_5=1.0



(a) Shock: $\varepsilon wd > 0$ (b) Shock: $\varepsilon ws > 0$

Figure 8 - Fuel: response of domestic variables to global shocks - MLC condition

Note: Dotted lines correspond capture the 68% probability interval. Source: Prepared by the authors.

Table 14 – Fuel trade response: percentage change and elasticities. Prior: μ_5=1.0

Time interval	Δtot	Δx	Δm	Δtb	ηx	ηm	$\frac{\Delta tb}{\Delta tot}$	$\eta_x - \eta_m - 1$
Box A - shock ε^{wd}								
1^{st} year	-6.6%	-3.5%	9.5%	-0.5%	0.5	-1.4	0.1	1.0
cumulative to the 3rd year	-6.2%	-11.5%	35.2%	-21.9%	1.9	-5.7	3.5	6.5
Box B - shock ε^{ws}								
1^{st} year	-8.8%	-0.2%	1.9%	9.2%	0.02	-0.2	-1.0	-0.8
cumulative to the 3rd year	-12.2%	-18.6%	17.9%	-9.3%	1.5	-1.5	0.8	20

Notes: Δ refers to percentage variation against the pre-shock level. $\eta_x = \frac{\Delta x}{\Delta tot}$ and $\eta_m = \frac{\Delta m}{\Delta tot}$. The Marshall-Lerner is satisfied if the values reported in last two columns are positive.

Bibliography

Ali, Syed Zahid, and Sajid Anwar. "Anticipated versus unanticipated terms of trade shocks and the J-curve phenomenon." *Journal of International Money and Finance* 81 (2018): 1-19.

Antolín-Díaz, Juan, and Juan F. Rubio-Ramírez. "Narrative sign restrictions for SVARs." *American Economic Review* 108, no. 10 (2018): 2802-2829.

Arruda, Elano Ferreira, Antônio Clécio de Brito, and Pablo Urano de Carvalho Castelar. "Exchange Rate and Trade Balances in Brazil: A Disaggregated Analysis by Major Economic Categories." *International Journal of Economics and Finance* 14, no. 6 (2022): 1-62.

Arruda, Elano Ferreira, Pablo Urano De Carvalho Castelar, and Gabriel Martins. "The J-Curve and the Marshall-Lerner Condition: Evidence for net exports in the Southern region of Brazil." *Planejamento e Políticas Públicas* 52, no. 1 (2019): 17-48.

Arruda, Elano Ferreira, and Gabriel Martins. "Taxa de câmbio e exportações líquidas: uma análise para os estados brasileiros." *Nova Economia* 30 (2020): 111-142.

de Azevedo, Dalylla Soares, Elano Ferreira Arruda, Antônio Clécio de Brito, and Pablo Castelar. "Dynamic elasticities of the trade balance of Brazilian agriculture in relation to the exchange rate." *The International Trade Journal* 38, no. 2 (2024): 201-218.

Backus, David, Patrick J. Kehoe, and Finn Kydland. "Dynamic of the trade balance and the terms of trade: The j-curve?" *American Economic Review* 84. no. 1 (1994): 84–103.

Bahmani-Oskooee, Mohsen, Augustine C. Arize, and Ebere Ume Kalu. "US-South America trade and the J-Curve: An asymmetric analysis." *The World Economy* 45, no. 12 (2022): 3858-3872.

Bahmani-Oskooee, Mohsen, and Scott W. Hegerty. "The J-and S-curves: A survey of the recent literature." *Journal of Economic Studies* 37, no. 6 (2010): 580-596.

Bahmani-Oskooee, Mohsen, and Artatrana Ratha. "The J-curve: a literature review." *Applied economics* 36, no. 13 (2004): 1377-1398.

Bekaert, Geert, Eric Engstrom, and Andrey Ermolov. "Macro risks and the term structure of interest rates." *Journal of Financial Economics* 141, no. 2 (2021): 479-504.

Bernanke, Ben S., Mark Gertler, Mark Watson, Christopher A. Sims, and Benjamin M. Friedman. "Systematic monetary policy and the effects of oil price shocks." *Brookings papers on economic activity* 1997, no. 1 (1997): 91-157.

Blanchard, Olivier Jean. "A traditional interpretation of macroeconomic fluctuations." *The American Economic Review* (1989): 1146-1164.

Boyd, Derick, Gugielmo Maria Caporale, and Ron Smith. "Real exchange rate effects on the balance of trade: cointegration and the Marshall–Lerner condition." *International Journal of Finance & Economics* 6, no. 3 (2001): 187-200.

Charnavoki, Valery, and Juan J. Dolado. "The effects of global shocks on small commodity-exporting economies: lessons from Canada." *American Economic Journal: Macroeconomics* 6, no. 2 (2014): 207-237.

Chen, Yu-chin, and Kenneth Rogoff. "Commodity currencies." *Journal of international Economics* 60, no. 1 (2003): 133-160.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. "Nominal rigidities and the dynamic effects of a shock to monetary policy." *Journal of political Economy* 113, no. 1 (2005): 1-45.

Cushman, David O., and Tao Zha. "Identifying monetary policy in a small open economy under flexible exchange rates." *Journal of Monetary economics* 39, no. 3 (1997): 433-448.

Dalla, Violetta, Liudas Giraitis, and Peter CB Phillips. "Robust tests for white noise and cross-correlation." *Econometric Theory* 38, no. 5 (2022): 913-941.

Drechsel, Thomas, and Silvana Tenreyro. “Commodity booms and busts in emerging economies.” *Journal of International Economics* 112 (2018): 200-218.

Enders, Walter. *Applied econometric time series*. John Wiley & Sons, 2008.

Ermolov, Andrey. “Time-varying risk of nominal bonds: How important are macroeconomic shocks?” *Journal of Financial Economics* 145, no. 1 (2022): 1-28.

Fernández, Andrés, Andrés González, and Diego Rodriguez. “Sharing a ride on the commodities roller coaster: Common factors in business cycles of emerging economies.” *Journal of International Economics* 111 (2018): 99-121.

Fernández, Andrés, Stephanie Schmitt-Grohé, and Martín Uribe. “World shocks, world prices, and business cycles: An empirical investigation.” *Journal of International Economics* 108 (2017): S2-S14.

Ferreira, Mauro Sayar, and André Cordeiro Valério. “Global shocks in emerging economies: An empirical investigation.” *Revista Brasileira de Economia* 76, no. 3 (2022): 315-348.

Ramos Filho, Hélio Sousa, and Maysa Evellyn Porfírio Ferreira. “A taxa de câmbio e os ajustes no saldo da balança comercial brasileira: uma análise setorial da curva J.” *Nova Economia* 26 (2016): 887-907.

Reis Gomes, Fábio Augusto, and Lourenço Senne Paz. “Can real exchange rate devaluation improve the trade balance? The 1990–1998 Brazilian case.” *Applied Economics Letters* 12, no. 9 (2005): 525-528.

Kilian, Lutz, and Helmut Lütkepohl. *Structural vector autoregressive analysis*. Cambridge University Press, 2017.

Kim, Soyoung. “International transmission of US monetary policy shocks: Evidence from VAR’s.” *Journal of monetary Economics* 48, no. 2 (2001): 339-372.

Kose, M. Ayhan. “Explaining business cycles in small open economies’: How much do world prices matter?’.” *Journal of International Economics* 56, no. 2 (2002): 299-327.

Leeper, Eric M. “Narrative and VAR approaches to monetary policy: Common identification problems.” *Journal of Monetary Economics* 40, no. 3 (1997): 641-657.

Lütkepohl, Helmut. *New introduction to multiple time series analysis*. Springer Science & Business Media, 2005.

Mendoza, Enrique G. “The terms of trade, the real exchange rate, and economic fluctuations.” *International Economic Review* (1995): 101-137.

Miranda-Agrippino, Silvia, and Giovanni Ricco. “The transmission of monetary policy shocks.” *American Economic Journal: Macroeconomics* 13, no. 3 (2021): 74-107.

Moura, Guilherme, and Sergio Da Silva. “Is there a Brazilian J-curve.” *Economics Bulletin* 6, no. 10 (2005): 1-17.

Park, Joon Y., and Peter CB Phillips. “Statistical inference in regressions with integrated processes: Part 1.” *Econometric Theory* 4, no. 3 (1988): 468-497.

Park, Joon Y., and Peter CB Phillips. “Statistical inference in regressions with integrated processes: Part 1.” *Econometric Theory* 4, no. 3 (1988): 468-497.

Ribeiro. M. P; Vasconcelos. C. F. V.; and Silva. C. Exchange variation and non-linear effect on exports: a sectoral analysis by technological intensity. 51º Encontro Nacional de Economia. Rio de Janeiro. RJ. Brazil. 2021.

Rocha, Francisco JS, Macos RV Magalhães, and Átila Amaral Brilhante. “A BVAR Note on the J-Curve and the Marshall-Lerner Condition for Brazil.” *International Journal of Economics and Finance* 16, no. 3 (2024).

Schmitt-Grohé, Stephanie, and Martín Uribe. “How important are terms-of-trade shocks?” *International Economic Review* 59, no. 1 (2018): 85-111.

Senhadji, Abdelhak S. “Dynamics of the trade balance and the terms of trade in LDCs: The S-curve.” *Journal of International Economics* 46, no. 1 (1998): 105-131

Shousha, Samer. “Macroeconomic effects of commodity booms and busts: The role of financial frictions.” *Unpublished manuscript* (2016).

Sims, Christopher A. “Macroeconomics and reality.” *Econometrica: journal of the Econometric Society* (1980): 1-48.

Sims, Christopher A., James H. Stock, and Mark W. Watson. "Inference in linear time series models with some unit roots." *Econometrica: Journal of the Econometric Society* (1990): 113-144.

Sims, Christopher A., and Tao Zha. "Bayesian methods for dynamic multivariate models." *International Economic Review* (1998): 949-968.

Uribe, Martin, and Stephanie Schmitt-Grohé. *Open economy macroeconomics*. Princeton University Press, 2017.

Zha, Tao. "Block recursion and structural vector autoregressions." *Journal of Econometrics* 90, no. 2 (1999): 291-316.

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CONFLITO DE INTERESSE

Os autores declaram não terem quaisquer conflitos de interesse.

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