ON A POSSIBLE CONTRIBUTION OF TRANSFINITE MATHEMATICS TOWARDS EURHYTHMY

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Knowledge never achieves completion. Any system of theories, however excellent, ends by generating anomalies and paradoxes. This statement is valid for philosophical systems, scientific theories or other forms of investigative knowledge. If one takes, say, the history of physics, there are many examples to illustrate the point, such as the Ptolemaic geocentric system, or Newton's mechanics, or the orthodox quantum theory. There is no methodology to create axioms and rules that remain forever valid, therefore it is capital that the history of science be concerned with controversies that surround this knowledge, including the socio-cultural environment where these axioms originated. From the epistemological viewpoint, it is useful to exploit how the reactions to the controversies were, how they were judged at the time when different hypotheses, theories and experiments arose – as well as how they are historically and scientifically evaluated in the present.

On the other hand, nature does not function arbitrarily, rather it is open to our rationality, and it is precisely this feature that allows us to know it, in better and better approximations. That nature is not arbitrary can be demonstrated also in modern mathematical applications, such as the theory of chaos: natural processes which are seemingly random, in a vast number of cases show themselves to possess a hidden regularity, which was inaccessible at first view. It is thus reasonable to assume that there is always progress in knowledge, even though we face the contingency of creating new theories from time to time. There is also an attenuating factor in this process of transformation of the scientific body: even if dogmas from the prevailing scientific canon are changed, usually a new theory accepts as a valid limit case the old theory. So, there is no abrupt revolution in the short term, but gradual reforms. Even though many philosophers of science maintain that no comparison is possible between paradigms that are incommensurable, it is doubtful whether the two systems really do not converse with each other. Old and new controversies will intermingle and contribute towards an epistemological deepening and mutual better understanding.

The above considerations hinge on our disagreement with the opposition, sometimes in radical tones, between realism and idealism, when the latter term is taken as synonymous of Plato's concept of ideas. One can say that every theory intended to explain the world is, in principle, a model, *i.e.* a representation of reality. Model building allows some forecasts, when the model is really good; historically speaking, explanatory models replace each other, but as noted above, in general something is retained out of the preceding models, originating a succession of more or less entangled models, permeating the advancement of knowledge. In the so-called Platonic world of ideas, the truth exists as well as the true idea – we may have limited access to this true world, while our models somehow probe such truth, though always in an incomplete way. In the well-known myth of the cave, Plato exploits the approximate discovery of the true idea, and his allegory adjusts well to even the most realist theories – always in a provisory form – that science adopts.

From this vantage point, we add our opinion that the history of the sciences has quite often corroborated that, in particular, mathematics is rooted in the activities of observing and interpreting reality. Accordingly, mathematics somehow does discover what already exists in the universe, even when theories are apparently invented out of the blue. Although the mathematical science has and fully uses freedom to propose hypotheses and axioms, possibly more independently than in other sciences, it is remarkable that even those theories that appear to be most abstract and devoid of experience, sooner or later they end up as practical applications in natural sciences like physics, chemistry, biology or other forms of knowledge.

We will then here advance some reflections involving on one hand the recent proposals relative to eurhythmy made by the Lisbon Group [Croca: 2010], and on the other hand a topic which has already stirred much horror among the mathematicians: the discoveries about infinity by Georg Cantor (1845 – 1918). What is to be investigated is whether also in the case of infinities, there can be some application to this newborn theory of physics. We shall proceed very carefully, since we do not have sufficient elements to make it operational.

Initially we recall that Cantor in his 1883 *Grundlagen* (*Foundations of a general theory of manifolds,* Note to Section 1) justifies his resource to different infinities by quoting Plato's dialogue *Philebus or the Highest Good.* There is in this dialogue an interesting passage (16 b,c,d) that bears upon our subject; let us hear Plato speak though Socrates:

> *... there is not, and cannot be, a more attractive method than that to which I have always been devoted, though often in the past it has eluded me so that I was left desolate and helpless…It is a method quite easy to indicate, but very far from easy to employ. It is indeed the instrument through which every discovery ever made in the sphere of the arts and sciences has been brought to light…The men of old, who were better than ourselves and dwelt nearer the gods, passed on this gift in the form of a saying. All things, so it ran, that are ever said to be consist of a one and a many, and have in their nature a conjunction of limitedness and unlimitedness.*

The elaboration of this thought in number theory was Cantorian mathematics' main contribution to science. The departure point that enabled Cantor to arrive at the transfinite, besides the already-mentioned Platonic inspiration, seems to have been St. Augustine, who in *The City of God* (Book 12, chap. 19) characterized the whole series of integer numbers as a real infinity, and not just a potential, or virtual one, as demanded by ancient philosophy, especially Aristotelianism.

Georg Cantor studied philosophy in Zurich, and then mathematics with Karl Weierstrass (1815 –1897) in Berlin, where he finished his doctorate in 1867. Until 1878 his works dealt with classical mathematics, and after that, he worked on the theory of infinite numbers. His original results led him to be severely attacked by orthodox mathematicians orchestrated by his great personal enemy, Leopold Kronecker (1821 – 1897), who defamed him and kept him from being published in prestigious mathematics journals, as well as tried by all means that he failed to get a university chair. Cantor's isolation due to such persecution plunged him in periods of depression, paranoia and nervous breakdown, but which he was initially able to overcome, going back to writing his major work, *Contributions to the founding of a theory of transfinite numbers* (1895 – 1897). Afterwards he suffered again several other mental crises, and ended up interned in a psychiatric hospital, where he died. Recognition of his work was late, and it still suffers attacks today – even though the famous mathematician David Hilbert (1862 – 1943) stated (1926) that "no one can drive us out of the Paradise created for us by Cantor".

Cantor calls a set (German "Mannigfaltigkeit", or "manifold", multiplicity) everything that is complete and determinate, even if infinite. A set constitutes a collection of objects of sensation, intuition or thought, and for him such a collection is related to the Platonic “idea”. Besides finding inspiration in Plato, and in the philosopher and theologian Augustine, he carefully studied Thomas Aquinas and Nicholas of Cusa, besides Giordano Bruno, who all also reflected on the matter of infinity. Basically, in his intellectual journey he took sides against Aristotle, for whom real infinity does not exist, since everything perceived by us is finite and limited, and from our limited sense perception derives the mind’s finitude.

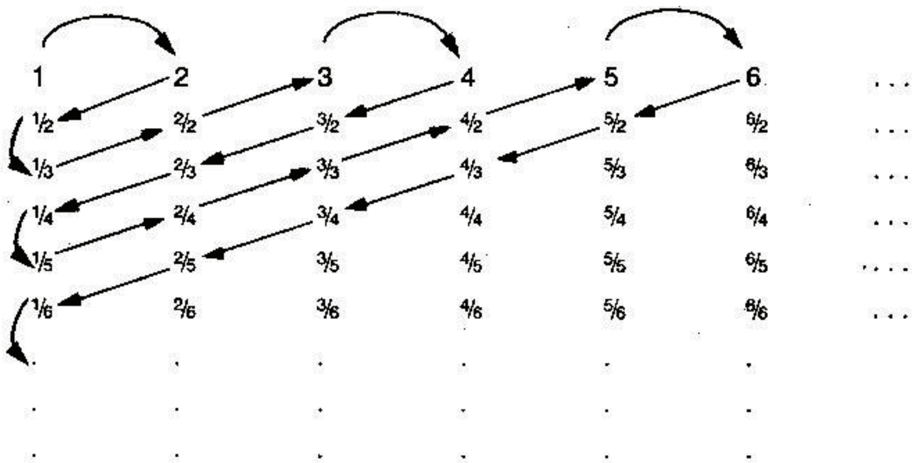
We know that in physics something as a light wave is not really infinite in time or space, though considering it as infinite might help disclose its approximate behavior, an approximation which is useful under certain circumstances. Indeed, only recently, by substituting the infinite waves which have been used since the 19th century in Fourier’s analysis by finite wavelets, as did the leader of Lisbon Group, José Croca, it was possible to create a truly causal theory for quantum phenomena which has revealed itself consistent. Are there infinite systems in our common practice? Even if we object to this on physical grounds, adopting the Aristotelian standpoint which admits of the sense limits imposed on space and time, we have not yet been able to verify whether real space is infinite or not, we can at most admit that “infinite” may stand for an approximation of an unreachable space finitude.

The same discomfort with infinities revealed in physics praxis has surrounded mathematical thought. Reformulating the question, mathematically speaking, is there a contradiction in assuming infinite sets to exist? Conceptually, the answer is negative: at least for Cantor and his adepts, we are entitled to consider infinities as an exact mathematical description. Richard Dedekind (1831 – 1916), Cantor’s close friend and correspondent, defined (*Essays on number theory,* 1888) as infinite the system which is similar to a part of itself, and then proved that there exist infinite systems by using the set S made up of the totality of things that can be object of his own thoughts. If *s* is an element of S, then the new thought *s'* that *s* can be an object of thought is itself an element of S. If we regard this as a transform $\Phi(s)$ of the element *s,* the transformation Φ has the property that the transform is part of S; certainly S’ is part of S, there are elements in S (e.g. the own ego) which are different from such thought *s'*, and therefore are not contained in S’.

The capacity of thought to think itself was used by Cantor in his second major publication on the transfinite, *Contributions to the founding of the theory of transfinite numbers* (1895 – 1897). In this one, he started with the set of natural numbers 1, 2, 3, 4…, which can be put in a one-to-one (biunivocal) relationship with the set of even numbers 2, 4, 6, 8…, even though the latter is contained in the former. Sets will be “equivalent” whenever one of them or part of it is thus related to the whole of the other set. This property is in turn connected with the linearization of the counting process, for a linear whole contains its parts, yet the attempt to linearize infinite sets confronts us with a paradox: the whole is equivalent to its parts, and this is the property that defines an infinite set, or as Cantor called it, a transfinite. Linearization can extend, therefore, the properties of a set until infinity, but in a fixed mode, which does not change the process in itself.

Incidentally, these properties of linearization reappeared more recently when the fractal theory originated in mathematics: through Mandelbrot sets geometrical figures are generated, so that when these are enlarged they are seen to reproduce the initial configuration. These fractals are also infinite sets, and they verify the statement that the whole is equivalent to a part of it. Once more mathematics and reality are combined, since fractals are the best description for a series of physical processes: the anatomy of biological networks, such as plant vessels or the nervous system; the geographical profile of the sea coastal zones; and many other practical applications.

In a letter (1885), Cantor says that the series 1, 2, 3... is a variable magnitude, which may increase without limits – it is a potential infinity. He then defined the "power" (Mächtigkeit), or "cardinality", of a set as a number that denotes a transformation measure: how many orders of abstraction differentiate a given set from another one. This was the consequence of Cantor's perception that the infinite natural numbers could be arranged in a one-to-one relation, not only with the infinite even or odd numbers, but also with the infinite fractions (rational numbers), containing integers both in the numerator and denominator. His reasoning was according to the following figure:



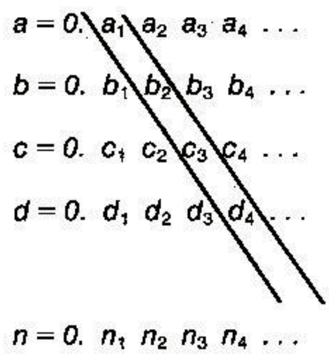
The demonstration by Cantor that infinite sets of rational numbers are countable and have the same power as the natural numbers is ingenious: in the preceding figure fractions are arranged in a matrix, such that the first1 line has all fractions n/1 (n are the natural numbers). The second line has all fractions n/2, the third one n/3, and so successively. A diagonalization process is then applied, starting with fraction 1/1, then going to 1/2, 1/3, 2/2, 3/1, etc. In the n-th step "n" elements have been picked, so that all possible fractions will be included. The result obtained is a one-to-one correspondence among the natural numbers of the first line and all possible existing fractions, which are therefore countable, even though there is an infinite number of them, as between 1 and 2, for example. There is also a one-to-one correspondence between the set of natural (or countable) numbers and many others, as the set of all square numbers, or also the set of all prime numbers.

However, since Ancient Greece we have known that there are numbers such as the square root of 2, which cannot be expressed by fractions, the so-called irrational numbers. Cantor was surprised to discover that the algebraic irrational numbers, i.e. those that can be constructed with ruler and compass (as √2, which can be constructed as the diagonal of a square whose side is equal to 1), though there is an infinite number of them, could also be put in a one-to-one correspondence with natural numbers. Cantor called all these different types of numbers "countable", pointing out that they all have the same generative principle, i.e. the same power: they can all be linearly ordered, and correspond to definite points on a line, where they can be counted using the sequence of natural numbers.

The sets of numbers described so far make the line be infinitely dense. However, there are "gaps", since we left out the transcendental, or non-algebraic, numbers such as π, the ratio between the circumference and its diameter. That was the key for Cantor to realize that there is more than one type (or power) of transfinite set. As in the case of √2, the number π cannot be expressed as a fraction, but more importantly, it cannot be constructed with ruler and compass, what would be possible if it could be defined through an equation similar to the one that defines the diagonal of a square; π can be given as the sum of an infinite series, which is in practice an approximation, but not an exact expression, thus it cannot be physically constructed. Therefore π is irreducible, it is something primitive and given, and though it is a very concrete relation between two values, the circumference and its diameter, it can be only idealized in the Platonic sense.

Taking into account the transcendental numbers, the infinity of countable numbers is not any more sufficient to hold the new set. The power of a set of real numbers, including the transcendental ones, is greater than the power of countable numbers, and so Cantor called the first one "non-countable". The first infinity is countable and has a cardinal number he called $\aleph_0$ (aleph-0), while the second infinity is non countable, with a cardinality of $\aleph_1$ (aleph -1).

To demonstrate that real numbers are non-countable, Cantor used a different process of diagonalization, as in the following figure:



The demonstration that the real number set has a higher power is through *reductio ad absurdum,* by admitting there is a way to order them (*i.e.* to put them in a one-to-one correspondence with countable numbers), such as in the figure above. Cantor chose to represent real numbers as an infinite series of decimals, periodic or not. Any real number is thus expressed with an integer part, followed by decimals – for example, 2 = 2.000000...; π = 3.141592...; √2 = 1.414286... in the figure we suppose that the infinite list contains in principle all real numbers (to simplify, the integer part was chosen as zero). The new diagonalization devised by Cantor is intended to construct a new number, starting with the integer part. For a new first decimal, one chooses any digit different from a_1; for the second one, any digit different from b_2; and so on. The new number is indeed different from any other in the infinite list: it is altogether different from the first one because its first decimal is different; it is also different from the second one in the list, etc. If the new number is added onto the original list, we apply the same procedure and create a newer number, which is not present in the modified list. This means that there is no counting process that passes through all real numbers, contrary to the initial supposition.

It is surprising that the power of the set of points on so dense a line associated to the real numbers is the same as any subset of the line, as for example in the interval from zero to one. It is also equal to the power of points in any dimension, as in the unit area or unit volume. The process of formation of non-countable infinities is thus non linear.

Extending his reasoning beyond these concepts, Cantor concluded that it is possible to create a transfinite number next to aleph-one, i.e. aleph-two, and in fact transfinite such numbers, so that there is no transfinite greater than all others. Each one will be non-countable and characterized by a power; as seen before, integers, rational and irrational numbers have a power equal to zero, and the real numbers a power equal to one. For successively greater transfinite numbers, such that A<B<C..., Cantor demonstrated that $2^A = B$, $2^B = C$, etc

The cardinality is associated to the size of a set, that is, the number of elements it contains. The arithmetic of transfinite cardinal numbers follows some rules which are different from those of finite numbers, such as:

i) If *a,b* are cardinals, with $b \geq a$, and at least one of them (*b*) is infinite, then $a + b = b + a = a \times b = b \times a = b$.

ii) $a < 2^a$, *a* being finite or infinite.

For Cantor what matters is the generating principle of the new number classes, and each one cannot be generated from a simple linear increase of the preceding series, contrary to how we can, say, form an integer greater than any other one, just by adding one unit, i.e. through counting.

There is still an unsolved problem in the history of transfinite numbers: the continuum – the set C of all real numbers is part of aleph-one, or is it exactly equal to aleph-one? As stated before, by mapping real numbers onto an infinitely long line, one can demonstrate that the set of these numbers contains as many elements as there are in a segment of the line. This makes any line section infinitely dense and with no gaps: between any two real numbers, there is an infinity (of cardinality aleph-one) of real numbers. It is exactly the presence of the transcendental numbers which fills up the gaps. The "continuum hypothesis" is that the real number set has a cardinality of aleph-one, as Cantor sustained a number of times, but there is no proof of that. In 1938, Kurt Gödel (1906 – 78) proved that it was not possible to demonstrate the falsehood of the continuum hypothesis, and 25 years later Paul Cohen (1934 – 2007) proved also the reciprocal, i.e. it cannot be demonstrated to be true by using the axioms of set theory. Cohen still conjectured (*Set theory and the continuum hypothesis*, 1966) that the continuum would in fact be much denser, and would have a cardinality greater than any aleph, its generation being a totally different process than generally assumed – but that remains an open question.

Besides what has already been presented, Cantor worked with the concept of ordinality. A set has an ordinal number which corresponds to the position this set occupies in a list whose first position is that of the sets with just one element, the second position is that of the sets with two elements, and so forth. In the case of finite sets, cardinality and ordinality coincide. For infinite sets, their cardinal does not coincide with the ordinal, and there are infinite distinct ordinals (positions) for the same cardinal.

A new arithmetic for transfinite ordinal numbers arises then, with peculiar commutative, associative, and distributive properties, such as:

- $\alpha + \beta \neq \beta + \alpha$; $\alpha.\beta \neq \beta.\alpha$
- $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$, $(\alpha.\beta).\gamma = \alpha.(\beta.\gamma)$
- $\alpha.(\beta + \gamma) = \alpha.\beta + \alpha.\gamma$

If ω designates the ordinal number associated to the cardinal aleph-zero (countable numbers), one can demonstrate that $\omega + \omega = \omega$, and also $\omega.\ \omega = \omega$.

About the ancient "problem of the one and the many", Cantor wrote to his friend Richard Dedekind about a contradiction if the multiple set is considered a member of the original "one" set. It is the famous paradox identified by Cantor himself, and later by the end of the 19^{th} century discussed by the mathematician Cesare Burali-Forti (1861 – 1931), better known in the following version under the name of [Bertrand] Russel´s (1872 – 1970) antinomy: the barber of a village where no man shaves himself, and where every man is shaved only by this barber, can he shave himself? This reminds of the type of contradiction that arises of statements like: "this sentence is false". That is not, however, what Cantor considers valid in his formulation of set theory, for him this mental trap can be avoided by considering that a multiplicity, even when infinite, must be thought of as something wholly new and different from its countable elements, and can only be thus adequately conceived.

The very human mind, or world of thoughts (*Gedankenwelt*), was the object of a mathematical demonstration offered to Cantor by Dedekind, who considered such a set as an infinite multiplicity (manifold) with all infinite alephs that could be conceived. This point is extremely important: human mind, in spite of its biological limitations - along the line pointed out by Aristotle in relation to potential infinity – has an infinite possibility to form new classes of transfinite numbers with ordered growing powers. This mental exercise is key to mathematically formulating human creativity, which manifests itself in all areas of knowledge, including artistic creation. Aleph-zero would then be a first "mode of thinking", whereas aleph-one is a "mode of thinking the mode of thinking". It is as if each transfinite were a quantum, or a monad: it is a unity that may have parts, but it behaves as a new being, which is more than the sum of its parts - it is not linearly reducible.

In the context that Cantor discussed these notions there was also a theological problem, which defied his religious faith, although he was not affiliated to any church. Contrary to what may be thought nowadays, when it is common to push religious considerations entirely out of the scientific practice, it was exactly theology that helped Cantor solve some formal problems of transfinite mathematics. For him, it became clear that the transfinite of real numbers is a creation belonging to this world and can be intelligible to mankind, whereas an "absolute infinity" lies in Plato's world of ideas, or is, theologically speaking, God's uncreated and exclusive attribute. This led him to admit that man can face real infinities, and thus the unity of the multiple, without contradicting himself – a multiplicity is consistent with a "set" provided of its own individuality. This power to be himself a creator allows man to solve problems by further creating new problems. If there were no problems to solve, neither inconsistencies that arise out of them, man would have no need for creativity, all of which is pertinent to the world of thoughts, as Cantor and Dedekind perceived.

The above theological argument has an important mathematical and scientific implication: there can be no complete axiomatic system, since sooner or later new axioms have to be created to solve the ensuing paradoxes of incompleteness. Such a feature fully agrees with reality, as a careful study of the history of physics shows, or moreover the history of any science demonstrates. Truth becomes a pathway to be uncovered and followed, not a final goal: the ultimate transfinite is not within the human reach, the absolute infinity can only be intuited but never attained. The ultimate truth is not human, it belongs to the absolute – whether it is a "God" as Cantor believed, or the universe itself, in a natural version. This is the trail of advancement of knowledge, which keeps us from embracing nothingness, the empty – as novelist Michael Ende well characterized in his fable (for children and adults alike) *The endless story*, where he portrays how the worst threat for mankind would be the loss of creativity, leading into the advancement of nothingness – the destruction of fantasy entails the substitution of the universe for a non-universe.

The attempts to transform the whole of mathematics in just logic and symbolism, divorced from reality, received an impulse with the formalization undertaken by Bertrand Russell and Alfred Whitehead at the beginning of the 20th century, in their work *Principia Mathematica.* The mathematician Kurt Gödel (1906 – 1978) counter-attacked this philosophical trend at the beginning of the 1930s, when he published his essay "On formally undecidable propositions of *Principia Mathematica* and related systems". He demonstrated therein that any formal system, to be free of contradiction (or "consistent") must be incomplete, i.e. open to the generation of new laws and axioms. In 1964, Gödel expanded that initial scope to oppose Alan Turing's (1912 –

1954) work on cybernatics, including his pretension to build “artificial intelligence” through a computer.

The general definition of algorithm advanced by Turing is well known. It is associated with the so-called “Turing machine” (1936), a sequential device which processes a few simple operations in a recursive way (implying a “mechanical”, countable process), just like the future digital computers would do. However, using a Turing machine is feasible exactly only for countable procedures, which accept a precise and closed definition to establish a calculation, an algorithm. When it is applied to problems such as, for example, defining real transcendental numbers, the machine eventually does not know whether to stop or not its processing, for there is no algorithm to define such numbers, or any non-countable infinity. The validation of the calculation in this case must be accomplished through external means, as Gödel demonstrated, based on Cantor’s previous work.

In other words, what distinguishes the functioning of the human mind from any computer language is that we are open to accept contradictions in the form of ambiguities, anomalies, paradoxes, metaphors, etc, which we solve and incorporate, whereas formal systems are closed and exclude such contradictions and inconsistencies, under the risk of not being able to proceed – and any additional attempt to provide a rule for the computer to deal with ambiguities will be short-lived, as it will stop again at a further contradiction.

As a matter of fact, the advancement of knowledge, which is more readily verified in the case of natural sciences, has been possible through the constant intrusion of anomalies, as shown in the history of science. Such anomalies are like non-linearities in a process which up to a certain point is well conducted and well described by a linear approximation. Though this approximation can be quite useful within certain conditions, it may however reveal itself fallacious when extended beyond those limits. The non-linear may even be linearized and provide satisfactory answers, yet one must be ready at some point to meet paradoxes derived from the adopted approximation.

A new question arises: if the very human mind works with discontinuities (quanta) in the process of creation, do the natural processes in the universe we inhabit also share the essence of distinct parts, which in the end form transfinite sets? For Cantor the answer was undoubtedly yes, both the continuous and discontinuous, the transfinite and the finite are two aspects of a unitary whole, to which we also belong. To exemplify the matter, would the so-called “space-time continuum”, as popularized by the relativity theory, be a “stratified” continuum, yet a non-linear one? If it were so, in hyperphysis when dealing in space-time with the subquantum level, would the substrate allow for linearization, and appear like some traditional substrate, as reality often seems to be linear in a larger scale?

The answer will not be a simple one. Linearization is, at most, a particular case, and it is liable to introduce distortions, which can be more or less relevant. Even in the mathematical domain, this is an interesting possibility, when one considers for example the class of functions that Karl Weierstrass (1815 – 1897) demonstrated to exist: continuous functions which are nowhere differentiable. Such a function cannot be linearized, not even in an infinitely small neighborhood of any of its points. The property of being continuous and non differentiable occurs when the function's graph shows edges or when it leaps, or also if it neither converges nor is defined. How does this impact on physical processes at the subquantum level?

Very peculiar properties of the subquantum medium were assumed as hypotheses in hyperphysis. The stronger hypothesis is that, consonant with the principle of eurhythmy, the organization of the subquantum medium that we call an akron (e.g. an electron) has a kind of sensory, its "theta wave" (or "empty wave"), with which the akron "feels" its external world. This would be a special type of sensor, whereby the eurhythmy property causes the akron to move along the direction where the intensity of the theta wave is greater, to preserve its existence, which is in turn what directly or indirectly we can observe. In spite of its peculiar appearance and the "sensor" metaphor, which could be considered Aristotelian and more appropriate for biology than physics, the principle of eurhythmy was able to be successfully applied to explain well-known natural phenomena, such as the principle of Fermat (minimum time for light propagation in different media), and Snell's law for light refraction, or more generally the principle of least action (Maupertuis and others), all resulting from the greater efficiency and harmony associated with the cited eurhythmical property.

In the hyperphysis associated with akrons and theta waves, the akron is "generated" or "emerges" out of the subquantum medium, and besides possessing an infinitely larger intensity, it has a longer mean lifetime, compared to its theta wave. Moving at its own velocity, the akron's mother-wave tends to disappear, returning the wave's energy to the subquantum medium. For the theta wave to persist, it must be regularly "revisited" by the akron (the "visitation hypothesis"), or else the akron is decoupled from its mother-wave and will generate new theta waves as it moves along. The akron supposedly does not lose energy when moving, and it behaves as an infinite energy reservoir, capable of generating an infinity of theta waves. To derive the expression for the intensity of the theta wave as "seen" by an akron, it is assumed that if it meets another theta wave whose intensity is much greater than its mother-wave, it will "feel" just the energy of the stronger wave. Reciprocally, if the mother-wave is stronger, it will continue following the mother- wave.

In an analogous manner one could suppose that a function describing the internal structure of an akron at the subquantum level should not be linearizable below a certain minimum distance from it, i.e. non-linearity becomes an indicative of the existence of internal structures, which may also explain its movement in the substratum. The energy of the pair is practically concentrated in the akron, but it is the theta wave that guides it, its energy is relatively insignificant (an estimate is that it is 10^{-54} times smaller). How can such small energy, distributed in an undulatory manner, help guide the akron to where its intensity is a maximum, unless there is an internal structure to the akron?

We do not possess a physical answer in the model for the question, but a formal way of putting it would be to ascribe to the akron the mathematical property of infinite cardinality, letting the akron be associated with some aleph, so that its energy content follows the infinite composition of transfinite intensities. Of course, we do not intend anything but an approximate description for certain subquantum conditions, where for the akron's intensity we would have something like $\aleph + \aleph = \aleph$. This all being just conjectures, we may as well suppose that the interactions among akrons also follow transfinite arithmetic, using either cardinal or ordinal numbers. Also for any theta wave, it may be considered as composed by a mother theta wave and other theta waves, forming a packet of wavelets.

How can we describe the resultant of several theta waves and their akrons that interact mutually, given that each interaction is in itself non linear? The mathematical difficulty that hyperphysis has encountered first led to a prudent traditional solution, which is the return to linear approximation, thereby acknowledging that the validity of the result is a limited one. In this way several hypotheses to determine the akron's velocity were studied, by varying the intensity of the theta wave from which the akron emerges, the type of medium, and its initial velocity. Maybe by using transfinite mathematics one could obtain an applicable tool, with the advantage that it would not have the philosophical flaws that linear approximations carry, and considering some similarities in non-linearity between Cantor's theory and the properties of hyperphysis. In particular, if the akron has an infinite intensity a and the theta wave a finite intensity b, we obtain $a + b = b + a = a.b = b.a = a$.

It is as if we said that the principle of eurhythmy is a manifestation of a kind of interaction resulting of infinite intensities, which lead to the predominance of new infinities. Though the description provided is only barely qualitative, the reasons for the parallel between transfinite mathematics and hyperphysis phenomena are due to considerations like those recapitulated below:

- Real infinity is something constant, it does not change by addition or subtraction of set elements, or to say it in different words, the whole is similar to a part of itself. By reproducing its parts, the whole is equivalent to its parts, and this property defines what an infinite set is. Likewise, the set formed by at least an akron and its theta wave seems to display a similar property.
- Linearization may infinitely extend properties of a countable set, but it does so in a fixed manner which does not alter the process itself. On the other hand, the process of infinity formation is a non linear one. Each transfinite is a unit, even if it has parts, it behaves as a new being which is more than just the sum of its parts. In a totally general way, nature does not seem fit for an exact linearization of its processes, which fact becomes evident when a linear description of a phenomenon breaks down at a certain point. This is exactly the case of quantum physics, where linearization is intimately coupled with a non causal description of phenomena.
- What is supposed to happen in the vicinity of an akron at the subquantum level has some analogy with the behavior of continuous functions which are neither nowhere differentiable nor linearizable in an arbitrarily small neighborhood.

Finally we should note that hyperphysis faces at one point the same problem that has confronted quantum theories up to now, that is, how to deal with infinite values which probably are a result of ignoring the internal structure of entities. For example, in classic theory when one considers the electron radius to be practically zero, then its self-energy tends to infinity – as in the case of several other entities that are approximately considered as "point particles". To cope with this problem, quantum theory introduced the renormalization concept, which corresponds grossly to subtract an infinite value from another infinite one, under certain conditions, leaving a finite result.

The infinite values might however have a different meaning, one that we don't know yet how to interpret. A first clue is that infinities may indicate that the adopted approximation of the model is no more usable, i.e. we have reached a scale where a significant action of an internal structure comes into play. If we insist on ignoring the structure, we will meet with infinite values, and knowledge will only advance when there is courage enough to pursue that internal morphology. The proposal advanced in 1985 by Winston Bostick (1916 - 1991) for a "chayah" (Hebraic for "living") still deserves to be remembered. Such "living" electron has a filamentary structure akin to the fusion plasma vortices, both those produced in laboratories and the natural ones observed in stars. It is possible that at such scale there is enormous energy liberation – similar to what nuclear energy represented in the past when one still assumed the nucleus to have a structure of only protons and neutrons, and conversion of matter into energy in the processes of nuclear fission and fusion showed energy levels much above those known at the time. Analogously the subquantum medium structure may show unsuspected energy magnitudes.

Secondly, from the formal point of view, the mathematics now used in the renormalization procedure may not be adequate, as it may falsify the behavior of functions whose value is so great that we consider them to be infinite. Conceptually, we may be doing something equivalent to a false infinity statement, such as $\omega - \omega = 0$ (or some other finite value), instead of $\omega - \omega = \omega$, i.e. renormalization may falsely avoid unavoidable infinities, which are ultimately evidenced. Renormalization would then be no more than the reintroduction of linearity in a process where the infinities are there to emphasize the non-linearity of a phenomenon. Will hyperphysis avoid this procedure?

We leave these questions in the air, as they result from some still very hypothetical thoughts, to those willing to work with hyperphysis and eurhythmy, and not satisfied with the mathematical foundations for this work.