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ORIGINAL ARTICLE

Regulatory impacts on investments by Pension Funds in Brazil

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ABSTRACT

This article compares the efficient investment frontiers in light of the new standard of allocative thresholds allowed for assets guaranteeing provisions established by Resolutions of the National Monetary Council (*Conselho Monetário Nacional* [CMN]) Nos. 4661/2018 and 4994/2022, verifying the probability of achieving returns that cover actuarial liabilities requiring a guarantee of minimum performance. This study innovates by assessing regulatory impacts on efficient Pension Fund (*Entidades Fechadas de Previdência Complementar* [EFPCs]) frontiers, comparing the results of risks and viable returns obtained by the new standards, using Conditional Value-at-Risk as a coherent risk measurement, as it meets the axiom of subadditivity. Furthermore, we provide measurements of the probability of achieving specific actuarial targets and of the portfolio generating a negative result. The national supplementary pension system recently went through a crisis, related to fraud and corruption schemes in the State-owned EFPCs triggered in 2016 through Operation Greenfield. As the main response to the current context, brought by the new normative acts, risk management processes were adopted and implemented and more refined Corporate Governance mechanisms were defined in the decision-making processes related to the investment policies adopted by an EFPC. Including the flexibilization of allocative thresholds. An impact of this research is to provide theoretical support for the pension sector, in light of macroeconomic contexts possibly marked by lower interest rates, in addition to assessing the practical implications of changes proposed in the new normative resolutions. Especially because EFPCs have systemically relevant actuarial liabilities. The methodology involved conditional optimization of portfolios using Asset-Only Assets and Liabilities Management (ALM) models. Despite the flexibility of new standards, there are no differences in the returns potentially obtained, given the overlapping of efficient frontiers of models in each standard. It was found that the unrestricted model showed higher returns with substantially lower volatility when compared to restricted models, pointing out that portfolios with fewer legal constraints can generate less exposure to EFPC net worth, something extremely important for defined benefit plans.

Keywords: Pension Funds, Assets and Liabilities Management, asset management, defined benefit plans.

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Impactos regulatórios nos investimentos de Entidades Fechadas de Previdência Complementar no Brasil

RESUMO

Este artigo compara as fronteiras eficientes de investimentos diante do novo padrão de limites alocativos permitidos para ativos garantidores de provisões estabelecidos pelas Resoluções do Conselho Monetário Nacional (CMN) ns. 4.661/2018 e 4.994/2022, verificando a probabilidade de atingimento de retornos que cubram os passivos atuariais que necessitam de garantia de desempenho mínimo. Este estudo inova ao avaliar impactos regulatórios em fronteiras eficientes de Entidades Fechadas de Previdência Complementar (EFPCs), comparando os resultados dos riscos e retornos viáveis obtidos pelas novas normas, utilizando o Conditional Value-at-Risk como medida coerente de risco, por satisfazer o axioma da subaditividade. Ademais, fornecemos medidas de probabilidade de atingimento de metas atuariais específicas e de o portfólio gerar resultado negativo. O sistema nacional de previdência complementar passou recentemente por uma crise, ligada a esquemas de fraude e corrupção nas EFPCs estatais deflagrados em 2016 por meio da Operação Greenfield. *Como principal resposta ao contexto atual trazida pelos novos atos normativos, foram adotados e implementados processos de gestão de riscos e definidos mecanismos mais apurados de Governança Corporativa nos processos decisórios vinculados às políticas de investimentos adotadas por uma EFPC. Incluindo a flexibilização dos limites alocativos. Um impacto desta pesquisa é fornecer subsídios teóricos para o setor previdenciário, à luz de contextos macroeconômicos eventualmente marcados por juros mais baixos, além de avaliar as implicações práticas das alterações propostas nas novas resoluções normativas. Especialmente porque EFPCs têm passivos atuariais sistemicamente relevantes. A metodologia envolveu a otimização condicionada de carteiras usando modelos de Assets and Liabilities Management (ALM) Asset-Only. Apesar da flexibilização das novas normas, não há diferenças em retornos potencialmente obtidos, haja vista a sobreposição das fronteiras eficientes dos modelos de cada norma. Constatou-se que o modelo irrestrito apresentou maiores retornos com volatilidade substancialmente inferior quando comparada aos modelos restritos, indicando que portfólios com menores restrições legais podem gerar menor exposição ao patrimônio das EFPC, algo extremamente importante para planos do tipo benefício definido.*

Palavras-chave: *Entidades Fechadas de Previdência Complementar, Assets and Liabilities Management, gestão de ativos, planos de benefício definido.*

1. INTRODUCTION

According to the National Supplementary Pension Superintendency (*Superintendência Nacional de Previdência Complementar* [PREVIC]), a federal agency responsible for monitoring and supervising the national supplementary pension system in Brazil (excluding Open Entities, which are supervised by the Private Insurance Superintendence [SUSEP]), a Pension Fund (*Entidade Fechada de Previdência Complementar* [EFPC]) is an institution whose exclusive purpose is the administration of pension benefit plans for specific groups of public, private, or mixed companies, and they are not available for free membership by the public as a whole. Constituted under the aegis of Art. 35 of Complementary Law n. 109 (CL n. 109, 2001), and organized by companies, associations, or professional entities, the EFPCs are non-profit and aim to ensure beneficiaries (employees, dependents, or associates) additional income to the retirement resources provided by the General Social Security Regime, in addition to providing insurance protection against unwanted risks (e.g. disability or death).

The funding of an EFPC comes from the resources of participating employees and employers (sponsors), as determined in Art. 6 of Complementary Law n. 108 (CL n. 108, 2001). Such values are recognized in mathematical provisions (liabilities) of benefit plans and allocated to guarantee assets, generating financial returns for participants. Furthermore, it is noteworthy that the plans established by associations and professional entities are funded solely by participants, with no sponsor (Oliveira et al., 2017).

Legally, there are three types of benefit plans in Brazil: (i) Defined Benefit (DB), in which the values of future benefits to be paid are determined *ex ante* upon membership. Contributions may vary over time so that the initially set amount is reached, for the lifetime of retirement. (ii) Defined Contribution (DC), whose benefits are established *ex post*, at the time of retirement, as a function of the amount of contributions made (defined previously by the participant) and the income earned from individual accounts (Josa-Fombellida & Rincón-Zapatero, 2012). Finally, (iii) Variable Contribution (VC), with a hybrid characteristic between DB and DC.

Each modality has risks inherent to each counterparty in the event of an actuarial deficit or surplus of an EFPC.

In particular, the National Supplementary Pension Council (*Conselho Nacional de Previdência Complementar* [CNPC]), through Art. 14 of Resolution CNPC No. 30/2018, recommends that the obligation to settle actuarial deficits determined in DB plans falls mainly on the sponsor (Azambuja & Campani, 2022; Rodrigues, 2006), except if the sponsor is classified as State-owned, due to the legal prohibition of § 3 of Art. 6 of CL n. 108 (2001). On the other hand, if deficits are found in DC plans, the participant will bear the result, having a benefit decrease (Dong & Zheng, 2019; Sun et al., 2016).

The EFPCs are large institutional investors by nature. According to the latest PREVIC Statistical Report available (1st quarter of 2023), the EFPCs in Brazil currently total R$ 1.197 trillion in guarantee assets, equivalent to 12.1% of the national gross domestic product (GDP) during the year 2022. Out of the total number of supervised entities, only 135 were in surplus, while 24 are in technical balance, and 115 are in deficit. As they deal with social security benefits, these entities have strict rules for guarantee assets allocations. The regulation of the National Monetary Council (*Conselho Monetário Nacional* [CMN]) that governs the way in which EFPCs can invest their respective funds is Resolution CMN No. 4661/2018, revoked by Resolution CMN No. 4994/2022.

Resolution CMN No. 3792/2009, the old regulation, was in force for 8 years and 7 months and underwent a series of improvements that resulted in Resolution CMN No. 4661/2018. This update occurred after a peculiar context in the Brazilian economy, with successive cuts in the base rates, historically marked by high levels (Oliveira et al., 2017).

At the same time, some of the country's main pension funds have recently gone through a strong institutional crisis, linked to fraud, corruption, and embezzlement schemes in State-owned EFPCs, triggered in 2016 through *Operação Greenfield* (a task force coordinated between the Brazilian Federal Police and the Brazilian Federal Prosecutor's Office).

As the main practical response to that institutional context, risk management processes were adopted and implemented and more refined corporate governance mechanisms were defined in decision-making processes linked to investment policies adopted by an EFPC, via the edition of the 2018 normative act.

Table 1 shows the application thresholds of the guarantee assets permitted by Resolutions CMN Nos. 3792/2009, 4661/2018, and 4994/2022, by segment and class. In general, the main changes introduced in the 2022 standard do not substantially modify the allocation thresholds by class of guaranteeing asset, when compared to Resolution No. 4661/2018. The main changes make textual adjustments and focus on topics such as payment of performance fees and concentration by issuer, among others. Regarding the asset classes used in this study, there was no change in the maximum investment thresholds.

Table 1
Percentage allocation thresholds per class of guaranteeing asset in relation to plan resources – Resolutions CMN Nos. 3792/2009, 4661/2018, and 4994/2022

Asset Classes		3792	4661/4994	Description by Class and Thresholds
Fixed Income ***(Renda Fixa* [RF])**	Public Securities	**100%**	**100%**	Federal securities debt
		80%	**20%**	State and municipal securities debt[1]
	Investment Funds	**100%**	**100%**	Funds linked to domestic federal debt securities
		100%	**80%**	RF index funds
		20%[1]	**20%**	FIDC and FICFIDC quotas
	Others[1]	**80%**	**80%**	RF assets issued with financial institutions, publicly-held joint stock company
		80%	**20%**	Assets issued with non-bank financial institutions and credit unions
		80%	**20%**	Bonds of multilateral organizations issued in Brazil
		80%	**20%**	Debentures issued by privately held companies
		20%	**20%**	CCB, CCCB, CPR, CDCA, CRA, and WA
Variable Income ***(Renda Variável* [RV])**	Stocks	**70%**[2]	**70%**	Shares, bonuses, and subscription receipts, securities deposit certificates and funds referenced in shares of a listed company that ensure differential governance practices.
		60%[2] **50%**[2] **45%**[2] **35%**[2]	**50%**	Shares, bonuses, and subscription receipts, securities deposit certificates and funds referenced in shares of listed companies that are not in a special segment.
	Others	**-**	**10%**[3]	Brazilian Depositary Receipts (BDR) classified as level II and III
		3%	**3%**	Physical gold certificates traded on commodity and futures exchanges

Table 1
Cont.

Asset Classes		3792	4661/4994	Description by Class and Thresholds
Structured[4]	Investment Funds	20%	15%	FIP
		10%	15%	FIM and FICFIM
		-	15%	Funds classified as "Stocks – Access Market" (Resolution No. 4661), or Investment Funds in Emerging Companies.
	Others	-	10%	Structured Operations Certificates (Certificados de Operações Estruturadas [COE])
Real Estate[5]	Investment Funds	10%[5]	20%	FII and FICFII
	Others	20%[5]	20%	Certificates of Real Estate Receivables (Certificados de Recebíveis Imobiliários [CRI]) and Real Estate Credit Notes (Cédulas do Crédito Imobiliário [CCI])
Participant Operations		15%	15%	Personal loans and real estate funding
Investments Abroad	Investment Funds	10%	10%[6]	FIS and FICFIs classified as "RF – External Debt"
		10%	10%	Index funds from abroad traded on the Brazilian stock exchange
	Open Condominium Funds	-	10%	Funds with the suffix "Investment abroad" – that invest at least 67% of the P.L. in FI shares abroad
	Others	10%	10%	Brazilian Depositary Receipts (BDR) classified as level I
		10%	10%	Financial assets abroad belonging to the portfolios of Brazilian funds, not previously foreseen

Note: *CCB = Bank Credit Bills; CCCB = Bank Credit Bills Certificates; CDCA = Agribusiness Credit Rights Certificates; CPR = Rural Product Certificates; CRA = Agribusiness Receivables Certificates; FICFIDC = Funds of Credit Rights Investment Funds; FICFII = Funds of Real Estate Investment Trusts; FICFIM = Funds of Hedge Funds; FICFIs = Funds of Investment Funds; FIDC = Credit Rights Investment Funds; FII = Real Estate Investment Trusts; FIM = Hedge Funds; FIP = Equity Investment Funds; FIs = Investment Funds; P.L. = Equity; WA = Agricultural Warrants.*
Asset classification followed the criteria of Resolution No. 4661. When the respective asset and class diverge from Resolution No. 3792, these points will be duly indicated in the following observations.
(1) The combination of assets marked with this marking must be a maximum of 80% of the plan's resources (FIDCs and FICFIDCs are disregarded only for Resolution No. 4661).
(2) The asset class in the RV segment in Resolution No. 3972 only allowed investment in shares listed on B3, with application threshold restrictions according to the issuers' governance classification: 70% (Novo Mercado); 60% (Level II); 50% (Bovespa Mais); 45% (Level I); 35% (open companies not mentioned, and/or index fund shares referenced in shares admitted to trading on the stock exchange). Additionally, up to 25% of the plan's resources could be invested in bonds and securities issued by Special Purpose Vehicle (Sociedade de Propósito Específico – SPE).
(3) The new Resolution No. 4994 establishes that the acquisition of BDRs backed by index fund shares is also authorized via a managed portfolio, a portfolio of their own, or investment fund.
(4) Assets in the Structured segment cannot exceed 20% of the plan's resources (both resolutions). It is worth noticing that in Resolution No. 3792, FIIs were classified in the Structured and non-Real Estate segment, as well as CRIs, CCIs.
(5) The application threshold in the real estate segment was a maximum of 8% in Resolution No. 3792 considering only the following assets: I – real estate projects; II – rental properties for income; and III – other properties.
(6) The new Resolution No. 4994 established the possibility of direct purchase of federal public debt securities.
Source: *Prepared by the authors.*

Therefore, the use of robust techniques (Assets and Liabilities Management [ALM]) to predict financial flows in future scenarios that demonstrate mismatches between assets and liabilities is increasingly relevant (Gutierrez et al., 2019). In addition to these instruments being able to reveal efficient investment portfolios, it is possible to measure long-term insolvency probabilities, with the aim of mitigating possible future deficits with decisions in the present. Such mechanisms incorporate demands from the regulator itself and the other stakeholders engaged in maintaining the technical solvency of the EFPCs. Pachamanova et al. (2017) use ALM for determining the ideal investment strategy in a pension fund, finding financial returns capable of covering the institution's liabilities. Several recent ALM applications in the same context have been made (Andongwisye et al., 2018; Toukourou & Dufresne, 2018).

The main aim of this study was to verify whether the new standard of thresholds established by Resolution CMN No. 4661/2018 (Resolution CMN No. 4994/2022) was sufficient to achieve financial returns that cover the actuarial liabilities of EFPCs in Brazil, using an asset-only ALM model, through the comparison of efficient frontiers obtained by diversified portfolios.

2. THEORETICAL FRAMEWORK

2.1 Stochastic Programming Models and ALM

Although recent, the actuarial literature regarding ALM models is rich and prolific. As it is a technique with wide flexibility and a spectrum of applications to companies from various sectors, it is essentially applicable to operational contexts of insurance companies and banks, precisely because it captures exposures to subscription, credit, liquidity, and market risks (Duarte et al., 2017). The first commercial application was carried out in the context of the activities of a Japanese insurance company (Cariño et al., 1994).

Leibowitz et al. (1992) make a comprehensive review of the historical evolution of the development of such models, noticing that the emergence of the first instruments linked to ALM applied to pension funds were the Dedication Models (DM), intrinsically developed in an economic environment marked by high interest rates. Bhat (2020) states that there are 4 basic categories of ALM modeling: (i) single-period static models; (ii) single-period stochastic models; (iii) multi-period static models; and (iv) multi-period stochastic models. Bhat (2020) argues that the creation of Immunization Models (IM) had as its main aim portfolio management in scenarios with variations in interest rates. Decades later, with computational development, DMs were refined to adapt portfolio management in scenarios with decreasing interest rates (Waring & Whitney, 2009).

It must be understood that integrated ALM management is a long-term issue, whose intertemporal dynamics imply that essentially deterministic models are limited to deal with the behavior of actuarial parameters and macroeconomic variables, which are stochastic in nature (Saad & Ribeiro, 2004). So, the main aim of the DM models was to obtain portfolios traditionally marked by RF securities, facilitating a less costly and simplified management of invested resources, since the maturity of the securities was linked to the same maturity dates as the liabilities, more applicable to high interest conjectures (Ryan, 2014).

Gutierrez et al. (2019) assess a Chilean DC-type pension fund, to search for investment alternatives that offer various risk-return profiles. Consequently, it is notable how the development of more complex and robust techniques, such as stochastic programming (SP) or stochastic linear programming (SLP) methods, gained prominence in the corporate management process, as they incorporated more complex restrictions aligned with the reality of institutional investors (Hosseinzadeh & Consigli, 2017).

Josa-Fombellida and Rincón-Zapatero (2012) address the issue of resource allocation in a pension fund whose plan characteristic is DB. The interesting point in this case is that the approach is focused on Brownian uncertainty and the variational jumps of both benefits and assets are of the Poisson type. Ferstl and Weissensteiner (2011) suggest the multi-period SLP approach, which incorporates more realistic factors (e.g. a greater number of risky assets with transaction costs and taxes). While financial returns are calculated based on a first-order autoregressive vector, VAR(1), also incorporating coherent risk measurements such as Conditional Value at Risk (CVaR), which is minimized as optimal structures for investment portfolios are achieved.

In addition to incorporating interest levels on fixed income assets, it is possible to analyze a vast development of papers using the mean-variance (M-V) model, incorporating very diversified portfolios (Saad & Ribeiro, 2006; Zhang & Chen, 2016). Pan et al. (2018) use Geometric Brownian Motion (GBM) to obtain stochastic development of liabilities. In this context, considering the M-V issue, and using the Heston Model to model risky assets, the authors apply the Hamilton-Jacobi-Bellman (HJB) equation to derive closed expressions for the optimal investment strategy and efficient frontier. In their turn, Sun et al. (2016) focus on pension funds with DC-type plans, in which managers are able to invest in risk-free assets, and risky assets whose price follows jump diffusion processes. There is extensive recent ALM literature addressing DC plans in the accumulation phase (Li & Forsyth, 2019; Menoncin & Vigna, 2017; Wang & Li, 2018; Zeng et al., 2018).

2.2 Recent Contributions to the Literature

Although it is possible to identify numerous ALM approaches in various applications, particularly in the national context, the literature has few papers focused on EFPC. When it comes to verifying the operational performance of these entities, most studies focus on evaluating the variables that affect the performance of their investment portfolios, but using less robust techniques, such as Sharpe Ratio analysis and validation via hypothesis testing (e.g. Silva et al., 2020).

Valladão and Veiga (2008) made one of the most notable advances in the development of ALM in the

Brazilian scenario. Mainly because, to optimize the allocation of investments in a pension fund, they proposed the use of bootstrapping to generate scenarios with future returns in a stochastic multi-period model. Duarte et al. (2017) make another ALM application, however, their goal focuses on the impacts of adopting the regulatory thresholds imposed by the SUSEP for open supplementary pension entities.

More recently, Damasceno and Carvalho (2021) introduced ALM in an unexplored segment in Brazil: Social Security Regimes for Public Servants (*Regimes Próprios de Previdência Social* [RPPS]), a pension system for civil servants of federative entities. Furthermore, it is a pioneer in assessing whether the new legal investment thresholds imposed on RPPS are enough for these regimes to be capable of structuring diversified portfolios, providing financial returns that cover their respective actuarial liabilities. In this study, the authors use a mean-CVaR optimization model and verify that investment portfolios with regulatory restrictions for the allocation of amounts invested by asset classes have twice the risk exposure (volatility) when compared to portfolios constituted under the hypothesis of the absence of restrictive legislation. The study reveals that only RPPS classified at Levels III and IV of governance managed to achieve their respective actuarial goals, however, on the other hand, such occurrences were only observed in circumstances in which the risk of losses arising from investments was as high as possible.

Like this article, Oliveira et al. (2017) focus on the EFPCs. The authors build an ALM model based on multi-period SP, with data from a Brazilian pension fund governed by a defined benefit plan. The major contribution of this study lies in the proposition of an algorithm that incorporated several factors with potential impact on the cash flow of this entity. Specifically, the investment thresholds imposed by the old standard (Resolution CMN No. 3792/2009) were considered, also taking into account the regulatory restriction that the funding ratio (FR), the ratio between current assets and the present value of future liabilities, does not may be less than 1 in more than two consecutive years.

In light of that, the authors carried out the modeling of RF assets using the Cox-Ingersoll-Ross model, and the shares in which the EFPC invests have their prices generated by an GBM. The authors suggest that Brazilian pension fund managers should modify investment strategies, given the lower profitability of RF assets. Finally, the sensitivity of the EFPC insolvency probability in the various initial FR stands out, where only one of these initial values (1.672 – the highest of all) was capable of nullifying the entity's insolvency, respecting the 70% allocation threshold in RF and 30% in RV.

Paula and Iquiapaza (2022) contribute to the literature by analyzing investments from the EFPC perspective. However, unlike our study, the authors' goal was to assess the efficiency of various investment fund selection techniques for EFPC managers, specifically focusing on the analysis of 369 investment funds. To do this, they use various techniques and indicators, including CVaR and the Sharpe Index.

Therefore, the contribution of this article is to assess the profitability capacity of the assets guaranteeing the mathematical provisions of EFPC benefits in light of the new Resolutions CMN Nos. 4661/2018 and 4994/2022, in a possible scenario of lower interest rates in the Brazilian economy, which is historically marked by higher rates, using the methodology of Damasceno and Carvalho (2021). The study uses a database with an extensive and relevant sample period (10 years, from 2012 to 2022), incorporating several macroeconomic scenarios that impacted the performance of Brazilian EFPCs.

Additionally, by selecting non-specific assets, but rather consolidated indexes, we filled one of the gaps left by Paula and Iquiapaza (2022), who highlight the need for further research to focus on the incorporation of investment classes focused on *private equity* and investments abroad. Such classes were incorporated in this study.

3. METHODOLOGY

3.1 Technical Solvency Balance

This study applies the same asset-only ALM methodology implemented in Damasceno and Carvalho (2021). An ALM model aims to measure the mismatches of assets and liabilities of a DB supplementary pension plan based on Equation 1:

$$S_t = \frac{A_t}{P_t}, \quad (1)$$

where S_t represents the entity's technical solvency balance, A_t is the value of the plan's total assets, and P_t denotes the present value of the social security liabilities, all at the instant of time *t*, so that both A_t and P_t were determined via mark-to-market.

Thus, for $S_t \geq 1$, the plan is in surplus or, at least, in technical actuarial solvency balance. In turn, when $S_t < 1$, EFPC is considered a loss-making entity, eventually requiring adjustment measures (Rodrigues, 2006).

Also, Art. 30 of Resolution CNPC No. 30/2018 allows the possibility of using pricing adjustments to deduct accumulated deficit results.

To assess the technical solvency of the pension plan, Damasceno and Carvalho (2021) defined the rate of return needed at the instant of time t ($r_t^{\text{optimized portfolio}}$) so that a pension plan could be in actuarial balance:

$$r_t^{optimized\,portfolio} = \begin{cases} (1+{}_y i_t) \times (1+MA_t) - 1, & if\ A_t < P_t \\ MA_t, & if\ A_t \geq P_t \end{cases} \quad (2)$$

MA_t denotes the real annual actuarial target (i.e., without inflation) that the EFPC should achieve, and additionally, ${}_y i_t$ is considered an effective rate, such that $(1 + i_t) \times A_t = P_t$, i.e. it is the return obtained for $S_t = \mathbf{1}$. The authors highlight the need to annualize this rate, breaking it down into D years, also in real terms:

$${}_y i_t = (1+i_t)^{\frac{1}{D}} - 1 . \quad (3)$$

It was decided to apply multiple actuarial targets: 3%, 4%, 5%, 6%, and 7% real per year (p.y.). Although chosen arbitrarily, such values are based on the reality of the actuarial liabilities of some plans (Azambuja & Campani, 2022; Leal & Mendes, 2010).

3.2 The Optimization Problem: Risk Measurement and Allocation Thresholds

The issue of optimizing efficient investment portfolios for EFPC will be deployed with two restrictions: (i) the risk measurement defined by CVaR; and (ii) the allocative thresholds of investments by asset classes imposed by Resolution CMN No. 4661/2018.

Although Value at Risk (VaR) is the most widespread metric for assessing exposure to the risk of maximum expected loss, it does not satisfy the axiom of subadditivity (Artzner et al., 1999). So, Rockafellar and Uryasev (2000) argue that VaR measures only the most optimistic loss level, reflecting the lower threshold of the probability distribution of severities, completely ignoring the shape of the left tail. To overcome these limitations, they proposed CVaR, which has good properties (Rockafellar & Uryasev, 2002) and has been used in optimization issues (Gutierrez et al., 2019) in multiple decision contexts under uncertainty (Santiago & Carvalho, 2020).

CVaR represents the mathematical expectation of losses beyond a threshold in the probabilistic distribution of a portfolio's returns. Its representation (Krokhmal et al., 2001) is:

$$CVaR_\alpha(\omega,\zeta) = \zeta + (1-\alpha)^{-1} \int_{\xi \in \mathbb{R}} \left[f(\omega,\xi) - \zeta \right]^+ p(\xi)\, d\xi . \quad (4)$$

Thus:

$$VaR = \zeta_\alpha(\xi) = \inf\{\zeta | P(\xi \leq \zeta) \geq \alpha\} = -z_{(1-\alpha)}\sqrt{\omega^T \Sigma \omega}, \quad (5)$$

with $\boldsymbol{\omega}$ representing the vector of portfolio allocative weights, $\boldsymbol{\Sigma}$ a matrix of linear correlations between assets, and z_α is the $\boldsymbol{\alpha}$-quantile to the left of a Normal distribution (0,1). So, VaR and CVaR can be related using the following equation:

$$CVaR_\alpha(\omega,\zeta) = E\left[\xi \mid \xi \geq \zeta_\alpha(\xi)\right] = E\left[\xi \mid \xi \geq VaR\right] \quad (6)$$

According to Rockafellar and Uryasev (2000), it is possible to approximate CVaR numerically:

$$CVaR_\alpha(\omega,\zeta) \approx \zeta + (1-\alpha)^{-1} \sum_{s=1}^{S} \left[f(\omega,\xi_s) - \zeta \right]^+ p_s , \quad (7)$$

where the sum $\left[f(\omega,\xi_s) - \zeta \right]^+$ represents the excess losses incurred in the investment portfolio, beyond the fixed ζ threshold.

Analogously to Hernandez et al. (2021), mean-CVaR will be optimized:

$$\min \zeta + (1-\alpha)^{-1} \sum_{s=1}^{S} \left[f(\omega,\xi_s) - \zeta \right]^+ p_s \quad (8)$$

$$\text{s.a.} \ \sum_{s=1}^{S} \omega_i = 1$$

$$\sum_{s=1}^{S} \omega_i E[\xi_i] \geq R$$

$$z_s \geq f(\omega,\xi_s) - \zeta$$

$$z_s \geq 0.$$

To solve the optimization issue given by the set of Equations (8), it is needed to mathematically define the thresholds of Resolution No. 4661/2018 (considering any changes brought by Resolution No. 4994/2022). To do this, each threshold shown in Table 1 is inserted as:

$$\omega_n^{min} \leq \omega_n \leq \omega_n^{max}, \quad (9)$$

where $n = 1,2,3,...,N$ represents the proportion of each instrument available for asset allocation in accordance with current legislation.

3.3 Algorithm

The simulation algorithm was deployed by using the *R* software, version 4.0.4 (Peterson & Carl, 2018; Theußl et al., 2020). In this context, the generation of future scenarios will follow the same algorithm as Dempster et al. (2003). Thus, the simulated scenarios for each time point *t* will be given by the historical price series of each asset class estimated in a correlated manner. Therefore, Stochastic Differential Equations (SDE) models will be implemented according to Oliveira et al. (2017). In this way, all assets that are not fixed income will have their respective prices modeled by a GBM:

$$d\xi_{it} = \mu\left(\xi_{it},t\right)dt + \sigma\left(\xi_{it},t\right)dW_{it}, \quad (10)$$

with W_{it} following a Wiener process N(0, Δ) and $t < t + \Delta$.

Thus, in Di Domenica et al. (2007) the correlation of two assets is given by:

$$dW_i \cdot dW_j = \rho_{ij}dt \quad (11)$$

It is worth noticing that, for any assets *i* and *j*, we will have $\rho_{ii} = \rho_{jj} = 1$. If *i* represents some random future performance instrument, its value ($\xi_{i,t}$) is exactly determined by pricing that uses the GBM model, considering $\varepsilon \sim N(0, 1)$:

$$\xi_{i,t} = \xi_{i,(t-1)} e^{\left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma\varepsilon\sqrt{dt}} \quad (12)$$

3.4 Data: Determination of Annual Returns, Correlation between Classes and Simulations

Data regarding asset pricing were extracted from ANBIMA and Bloomberg. Thus, the historical quotations of certain assets or market indexes were used by considering ten years, i.e. from December 30, 2012 to December 30, 2022. With this panel, various events that affected the Brazilian economy were considered: Operation Lava Jato, impeachment of President Dilma Rousseff, and the coronavirus disease 2019 (COVID-19) pandemic.

Once we had the time series, data was annualized and the annual returns obtained for each asset class selected as an investment proxy to which an EFPC could apply its assets raised were calculated. Table 2 displays descriptive statistics of the distributions of historical returns for each selected investment class. The choice of such proxies was based on the investment criteria defined by Resolutions CMN Nos. 3792, 4661, and 4994, in order to cover all assets and thresholds displayed in Table 1.

Table 2

Descriptive statistics of the Asset Classes selected as proxies for investments permitted by Resolutions CMN Nos. 3792/2009, 4661/2018, and 4994/2022

Segment	RF		RV	Structured		Real estate	Investment abroad	
Asset Class	Federal Debt	Private Credit	Stocks	Private Equity	Hedge Funds	Real Estate Funds	RF – External Debt	Shares, BDRs and Others
Proxy Index by Segment	**IMAB**	**IDA_ GENERAL**	**IBOV Ibovespa**	**IBX**	**IHFA**	**IFIX**	**BGATT**	**MSCI**
Nominal Interest Rates	9.7%	8.1%	13.4%	18.4%	8.6%	10.2%	7.6%	9.7%
% on DI	3.0%	1.5%	6.5%	11.2%	2.0%	3.5%	1.0%	1.0%
Annual Returns								
Maximum	22.4%	9.9%	60.6%	105.5%	11.8%	38.6%	45.1%	53.2%
Average	5.0%	3.4%	8.5%	13.3%	4.0%	5.5%	2.9%	5.0%
Minimum	-19.2%	-5.7%	-36.3%	-67.8%	-8.2%	-28.6%	-41.6%	-48.8%
Standard deviation	8.2%	3.3%	17.5%	29.1%	3.9%	12.2%	17.3%	17.8%

Note: *BGATT = Bloomberg Global Aggregate Bond Index; IBX = Brazil Index 100; IDA-GENERAL = Anbima Debentures Index – General; IFIX = B3 Listed Real Estate Investment Funds Index; IHFA = Anbima Hedge Funds Index; IMAB = Anbima Market Index (NTN-B); MSCI = Morgan Stanley Capital International World.*
To calculate the values, the Selic discount rate of 6.5% and IPCA of 4.5% were considered. The IPCA reflects a value close to the market expectation of the Boletim Focus on 06/25/2021 for the end of 2022, allowing a more conservative approach given the constant volatility shown by the SELIC.
Source: *Prepared by the authors.*

This study applied the same method developed by Damasceno and Carvalho (2021) to generate returns in the Private Equity asset class. The authors argue that there is no measure or indicator capable of reproducing investment returns in this asset type. Therefore, they use the results of Minardi et al. (2017), in order to implement volatility adjustments (shocks) in the IBX historical series.

As expected, the asset classes that demonstrated the highest standard deviation measures were, in order: IBX (private equity), MSCI_W, IBOV (RV-Shares) and BGATT (external debt). On the other hand, it is worth noticing that greater volatility does not necessarily imply higher average returns, since the average returns observed for Real Estate Funds are higher than the values obtained for MSCI and external debt (BGATT), for instance.

Once the annual returns were defined, the next step consisted of building the correlation matrix of assets available for allocation of resources by the EFPC. Figure 1 summarizes the values observed. As expected, MSCI and BGATT are the classes most inversely correlated to investments linked to national assets, generating greater diversification in the entity's portfolio.

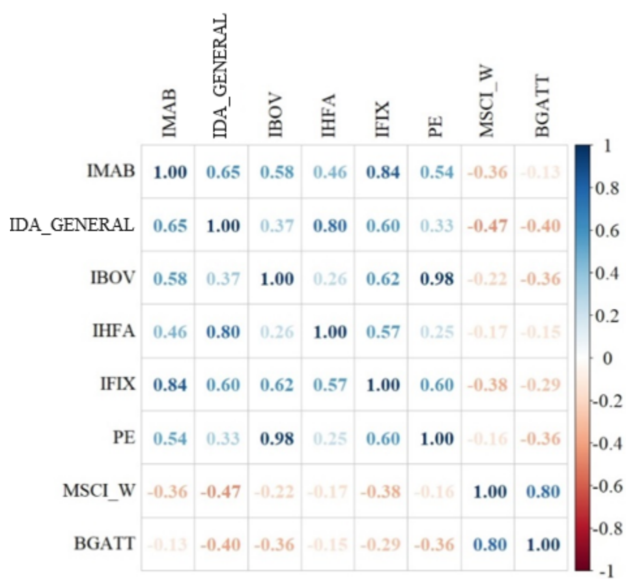

Figure 1 *Correlation observed between the investment classes considered in the study.*
Source: *Prepared by the authors.*

Thus, considering the changes and the respective thresholds proposed in each standard, three models were estimated: (i) Unrestricted Investment Model, created for comparative purposes, constituting a counterfactual scenario in which the EFPC could freely apply the plan's resources in any assets, without any legal restrictions; (ii) Investment Model according to Resolution CMN No. 3792/2009, with the restrictions imposed by the previous standard; and (iii) Investment Model in accordance with Resolutions CMN Nos. 4661/2018 and 4994/2022, representing the changes proposed by the most recent regulations. Figure 2 displays the annual risk and return results of the models optimized for each of the three scenarios, and their respective simulated portfolios.

For each model, 100 thousand portfolios were simulated. The choice of such a number is superior to that of Damasceno and Carvalho (2021), who already use a sufficiently large number and which also has a diversity of results.

4. RESULTS

4.1 Optimization Models: Efficient Frontiers and Portfolio Simulations

Resolution CMN No. 3792/2009 underwent several modifications until the publication of the most recent standards (Resolutions CMN Nos. 4661/2018 and 4994/2022). In general terms, it is observed (Table 1) relative flexibility between investment standards in the variable income segment. In particular, investments in shares are now limited to 70% and 50% of the EFPC's equity in shares of companies traded in special listing segments and outside such segments, respectively. The old standard had greater granularity, restricting the thresholds to 70% (Novo Mercado); 60% (Level II); 50% (Bovespa Mais); 45% (Level I); and 35% (others).

On the other hand, the most recent resolutions restricted the thresholds on investments in debentures and other assets related to private credit. The change in thresholds went from 80% to 20%. Furthermore, there was a reclassification of real estate investments, which were no longer classified along with structured assets. This resulted in an increased investment threshold for real estate funds from 10% to 20%. However, on the other hand, EFPCs now have restrictions on investing directly in the purchase of their own properties. Magnani et al. (2021) analyze the possible impacts of this restriction and provide further details on the territorial pattern of pension funds' real estate portfolio.

Also, there was a 5% decrease for investing in Private Equity funds, which was the asset with the highest annual return volatility in the database used, and this offered the highest premium over the DI return. In compensation, the allocation to hedge funds increased by the same magnitude (5%). Figure 2 displays the 100 thousand simulations of each model and their respective efficient frontiers. The various actuarial targets were inserted to highlight the ability to achieve each level.

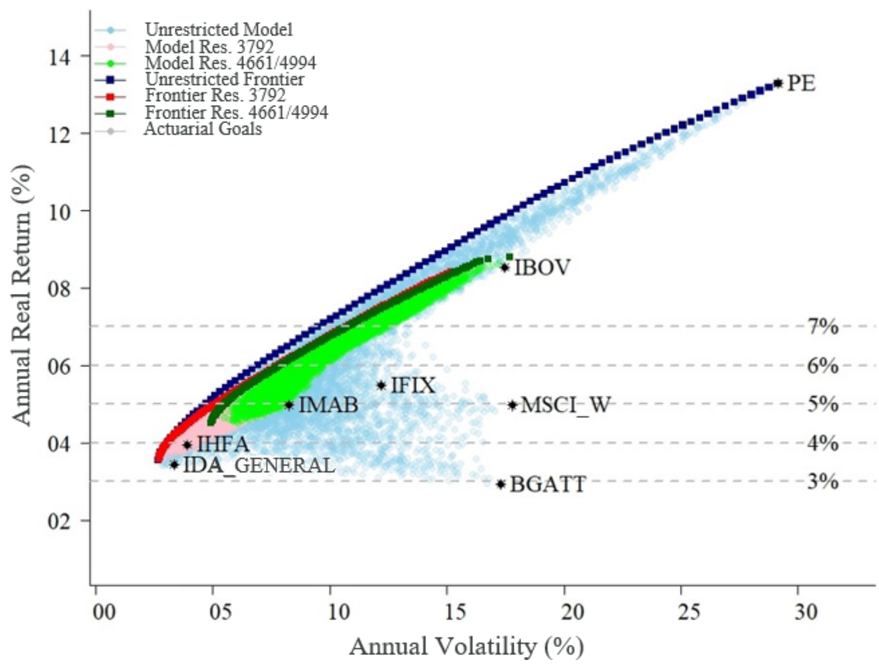

Figure 2 *Efficient Frontiers and their respective simulated random portfolios*
Source: *Prepared by the authors.*

Therefore, it is observed that the boundaries of normative acts virtually overlap for lower volatility levels. As expected, the higher allocation threshold on RV assets allowed the frontier of Resolutions Nos. 4661/2018 and 4.994/2022 to achieve higher returns for annual volatility levels above 12%. For lower volatility levels (between 5% and 10%), the threshold of Resolutions Nos. 3792/2009 had higher returns.

It is worth noticing that, in the simulations carried out, all actuarial targets were achieved by the models restricted by law. Figure 2 also reflects that unrestricted portfolios offer higher returns, without necessarily EFPC being exposed to greater risk. The evidence is that the efficient frontier of the unrestricted allocation model achieves all actuarial targets for volatility levels below 10%, unlike what occurs with normative models.

In turn, Figure 3 displays the allocation results by portfolio of the efficient frontiers found. The largest allocations in private credit are found in volatilities between 2% and 6% in the restriction model of Resolution No. 3792/2009. Resolution No. 4661/2018 imposed a legal decrease, forcing the optimization results to direct EFPC resources into federal public debt assets. Thus, at risk levels above 5.25%, the allocation to RV assets is considerably higher in the model under Resolutions Nos. 4661/2018 and 4994/2022, since given the allocation restrictions on shares by governance levels within the frontier of Resolution No. 3792/2009, the resources are redirected to public bonds at the cost of lower profitability.

In the unrestricted model, the optimization results generate, already at the initial risk levels, a greater allocation of resources in investments abroad, especially RF assets (proxy BGATT), with a large part in assets linked to private credit and equity investment funds and hedge funds. The highlight of this model occurs at volatility levels around 15%, when its frontier can achieve real returns close to 9% p.y., with portfolios consisting of PE assets, hedge funds, and investments abroad.

It is highlighted, through Figure 3 and Table 3, that in virtually no portfolio there was an allocation in the IFIX class, indicating that the change in the allocative threshold for the real estate segment did not significantly affect the results obtained, different from what the literature suggests (Bernardo & Campani, 2019). In restricted frontiers, given this same risk level, annual returns were 8.3%, suggesting that greater legal allocation thresholds in these classes can bring EFPC greater returns, without necessarily increasing exposure to risk. Table 3 displays the average allocation of efficient portfolios for each frontier.

Table 3
Average weights allocated by asset class in the optimized frontiers

PORTFOLIOS THAT	Reached 3%			Reached 4%		
Average Return	3.744%	3.762%	Min. Ret. > 4%	4.482%	4.469%	4.750%
Average Annual Volatility	0.028	0.028	-	0.038	0.039	0.052
Proxy/Models	**Unrestricted**	**3792**	**4661/4994**	**Unrestricted**	**3792**	**4661/4994**
IMAB	0%	0%	-	5%	5%	48%
IDA_GENERAL	82%	78%	-	38%	68%	20%
IBOV-IBOVESPA	3%	2%	-	1%	0%	7%
IBX	0%	0%	-	5%	8%	0%
IHFA	4%	9%	-	38%	10%	15%
IFIX	0%	0%	-	0%	0%	0%
BGATT	1%	1%	-	0%	0%	0%
MSCI	10%	10%	-	13%	10%	10%
PORTFOLIOS THAT	**Reached 5%**			**Reached 6%**		
Average Annual Return	5.466%	5.493%	5.507%	6.499%	6.492%	6.501%
Average Annual Volatility	0.057	0.0639	0.0663	0.082	0.0919	0.093
Proxy/Models	**Unrestricted**	**3792**	**4661/4994**	**Unrestricted**	**3792**	**4661/4994**
IMAB	12%	19%	38%	12%	35%	39%
IDA_GENERAL	0%	51%	20%	0%	27%	15%
IBOV-IBOVESPA	0%	0%	12%	0%	8%	16%
IBX	13%	16%	7%	23%	20%	15%
IHFA	59%	4%	13%	44%	0%	5%
IFIX	0%	0%	0%	0%	0%	0%
BGATT	0%	0%	0%	0%	0%	0%
MSCI	16%	10%	10%	21%	10%	10%

PORTFOLIOS THAT	Reached 7%		
Average Annual Return	7.532%	7.492%	7.495%
Average Annual Volatility	0.110	0.122	0.123
Proxy/Models	**Unrestricted**	**3792**	**4661/4994**
IMAB	13%	42%	33%
IDA_GENERAL	0%	3%	0%
IBOV-IBOVESPA	0%	25%	37%
IBX	34%	20%	15%
IHFA	27%	0%	5%
IFIX	0%	0%	0%
BGATT	0%	0%	0%
MSCI	26%	10%	10%
Other Values	**Unrestricted**	**3792**	**4661**
Returns: Min. / Max.	3.547% / 13.189%	3.592% / 8.369%	4.513% / 8.749%

Source: *Prepared by the authors.*

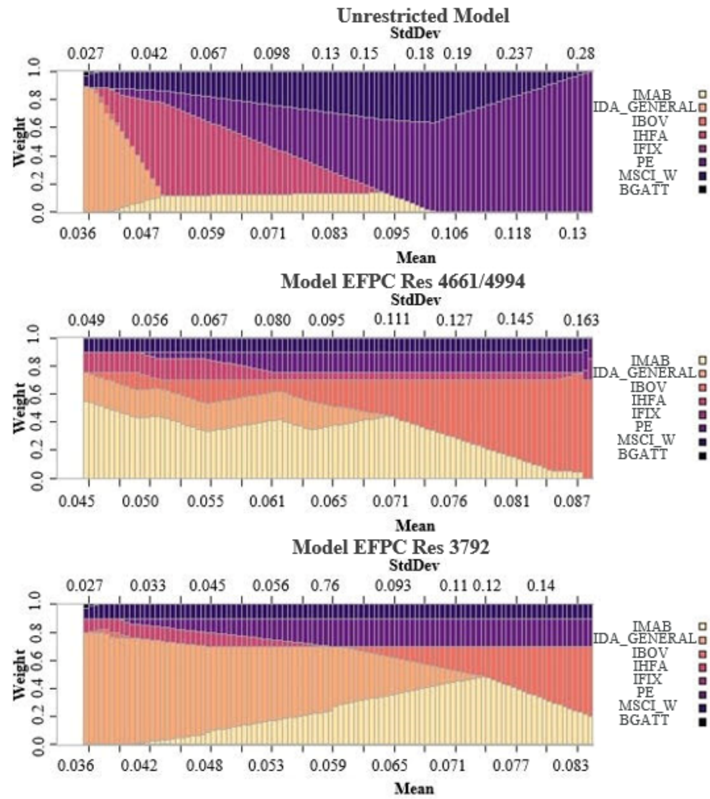

Figure 3 *Portfolio allocation on the efficient frontiers of simulated models*
Source: *Prepared by the authors.*

4.2 Probabilities of Positive and Negative Returns

To compare the behavior of the models and the future capacity of the estimated portfolios to achieve consistent returns over extended periods, 1,000 random scenarios were simulated, as done in Damasceno and Carvalho (2021), for six time horizons: (i) 12 months; (ii) 24 months; (iii) 36 months; (iv) 72 months; (v) 120 months; and (vi) 180 months. Thus, the accumulated annual returns were estimated by using the allocation weights per asset class existing in the optimal portfolios of each frontier. As example, for the actuarial target of 5% p.y., the allocative weights per class in the Unrestricted Model were: IMAB (11%); IDA_GENERAL (1%); IBOV-IBOVESPA (0%); IBX (9%); IHFA (65%); IFIX (0%); BGATT (0%); and MSCI (14%). Table 4 shows the descriptive statistics of cumulative annual returns.

Table 4
Descriptive statistics of accumulated returns for each actuarial target and time horizon

3% Target		Average			Standard Deviation	
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	3.6%	3.7%	4.8%	0.027	0.027	0.054
24 months	7.1%	7.7%	9.6%	0.041	0.041	0.077
36 months	11.2%	11.2%	14.6%	0.052	0.054	0.098
72 months	23.6%	23.9%	31.6%	0.080	0.082	0.161
120 months	42.8%	42.7%	57.7%	0.125	0.120	0.252
180 months	69.7%	71.8%	97.3%	0.173	0.179	0.380
Max.	134.3%	124.9%	241.0%	-	-	-
Min.	-5.4%	-5.1%	-14.7%	-	-	-

Table 4
Cont.

4% Target		Average			Standard Deviation	
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	4.2%	4.3%	4.8%	0.033	0.033	0.051
24 months	8.2%	8.3%	9.4%	0.047	0.051	0.074
36 months	12.8%	12.9%	14.0%	0.060	0.062	0.099
72 months	27.5%	28.0%	31.5%	0.100	0.098	0.156
120 months	49.8%	49.0%	57.7%	0.142	0.152	0.238
180 months	83.4%	83.2%	95.8%	0.225	0.225	0.374
Max.	171.3%	181.6%	220.7%	-	-	-
Min.	-7.6%	-6.6%	-15.3%	-	-	-
5% Target		**Average**			**Standard Deviation**	
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	5.2%	4.2%	5.1%	0.053	0.034	0.060
24 months	10.4%	8.4%	10.5%	0.078	0.049	0.087
36 months	16.4%	13.0%	16.5%	0.096	0.062	0.113
72 months	35.3%	27.3%	35.9%	0.162	0.100	0.183
120 months	65.6%	49.5%	65.9%	0.251	0.149	0.298
180 months	111.9%	83.4%	112.0%	0.406	0.230	0.465
Max.	272.4%	190.5%	355.3%	-	-	-
Min.	-12.6%	-9.8%	-14.7%	-	-	-
6% Target		**Average**			**Standard Deviation**	
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	6.2%	6.3%	6.0%	0.076	0.089	0.087
24 months	12.8%	12.4%	13.2%	0.115	0.129	0.128
36 months	19.6%	19.0%	20.6%	0.151	0.181	0.175
72 months	44.6%	44.7%	43.8%	0.274	0.292	0.295
120 months	82.6%	86.1%	83.0%	0.425	0.525	0.497
180 months	151.7%	151.2%	149.6%	0.717	0.811	0.811
Max.	533.3%	559.8%	544.2%	-	-	-
Min.	-17.7%	-33.9%	-25.8%	-	-	-
7% Target		**Average**			**Standard Deviation**	
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	7.5%	7.5%	8.3%	0.110	0.120	0.115
24 months	15.1%	16.0%	14.9%	0.171	0.187	0.188
36 months	24.9%	23.9%	24.4%	0.223	0.243	0.241
72 months	53.9%	53.8%	53.1%	0.412	0.427	0.430
120 months	104.9%	101.4%	104.0%	0.709	0.721	0.704
180 months	192.3%	190.7%	184.0%	1.280	1.291	1.282
Max.	1221.6%	948.6%	977.9%	-	-	-
Min.	-33.3%	-38.9%	-39.9%	-	-	-

Source: *Prepared by the authors.*

As expected, the distribution of returns began to increase as the time horizon increased. These results indicate that the probabilities of accumulated losses in EFPC portfolios tend to decrease, demonstrating consistency in the results obtained in the long term. Table 4 displays details of these estimated probabilities for different goals, model (unrestricted or subject to normative acts), and different measurement horizons.

Table 5
Probabilities of Achieving Actuarial Targets in 6 different future scenarios

Probability	of reaching the 3% target			of generating a negative return		
Time/Model	Unrestricted	3,792	4661/4994	Unrestricted	3,792	4661/4994
12 months	59.80%	60.70%	62.20%	9.70%	8.40%	16.70%
24 months	60.80%	64.50%	66.20%	3.80%	2.80%	9.90%
36 months	63.80%	62.60%	69.90%	0.90%	1.40%	5.70%
72 months	69.40%	67.80%	76.90%	0.00%	0.10%	1.50%
120 months	74.90%	74.30%	82.60%	0.00%	0.00%	0.00%
180 months	78.50%	80.70%	87.10%	0.00%	0.00%	0.00%
Probability	**of reaching the 4% target**			**of generating a negative return**		
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	51.40%	53.80%	53.60%	9.00%	9.00%	18.10%
24 months	49.20%	51.30%	54.20%	3.50%	4.10%	9.30%
36 months	51.10%	50.30%	54.10%	1.10%	1.70%	6.90%
72 months	52.80%	56.10%	61.80%	0.10%	0.00%	1.20%
120 months	52.20%	51.10%	63.90%	0.00%	0.00%	0.40%
180 months	53.50%	53.00%	64.30%	0.00%	0.00%	0.00%
Probability	**of reaching the 5% target**			**of generating a negative return**		
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	49.30%	39.00%	47.90%	17.10%	10.80%	20.40%
24 months	47.80%	36.90%	46.90%	7.50%	4.40%	10.70%
36 months	50.60%	29.70%	49.50%	2.80%	0.80%	5.80%
72 months	49.10%	24.70%	51.70%	0.40%	0.20%	1.60%
120 months	49.60%	18.20%	50.50%	0.00%	0.00%	0.30%
180 months	51.40%	14.50%	49.50%	0.00%	0.00%	0.00%
Probability	**of reaching the 6% target**			**of generating a negative return**		
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	47.20%	46.90%	47.90%	21.40%	24.20%	25.50%
24 months	49.60%	46.90%	49.90%	12.90%	14.90%	14.90%
36 months	47.10%	47.80%	50.40%	7.50%	14.40%	11.30%
72 months	48.10%	49.70%	48.30%	1.60%	4.40%	4.00%
120 months	48.80%	50.20%	46.10%	0.30%	1.30%	1.40%
180 months	50.70%	50.00%	48.10%	0.00%	0.40%	0.30%
Probability	**of reaching the 7% target**			**of generating a negative return**		
Time/Model	**Unrestricted**	**3,792**	**4661/4994**	**Unrestricted**	**3,792**	**4661/4994**
12 months	46.20%	47.70%	49.20%	26.20%	27.20%	24.80%
24 months	46.30%	47.90%	45.80%	18.70%	19.40%	22.30%
36 months	50.90%	46.00%	49.30%	11.40%	14.30%	14.60%
72 months	48.60%	48.20%	46.70%	4.80%	7.10%	7.30%
120 months	45.90%	46.30%	47.90%	1.20%	3.00%	2.90%
180 months	47.10%	45.80%	44.40%	0.20%	0.90%	1.30%

Source: *Prepared by the authors.*

In general terms, for all models, the simulations of future scenarios showed that pension funds have the feasibility of creating portfolios capable of reaching or exceeding 3% and 4% actuarial targets, respectively, with probabilities equal to or greater than 49.2%. Also, for the 3% and 4% targets, the probabilities of achieving the actuarial goals were substantially higher for the Resolution CMN No. 4661/2018 when compared to the other models, as the set of allocation weights chosen for the simulations in this case had to be that providing the lowest possible return on the efficient frontier of such model (i.e. 4.5127%) while the sets chosen for the unrestricted model and the Resolution No. 3792 Model generated, respectively, annual returns of 3.5472% and 3.5915% (simulated actuarial target of 3% p.y.), and 4.0391% and 4.0302% (simulated actuarial target of 4% p.y.).

However, it is worth highlighting that, for targets such as 6% and 7%, restrictive models by law make the EFPC portfolio substantially more exposed to losses. Especially in the short term (12 months). In the example for the 6% actuarial target, the probability of the Unrestricted Model generating a negative return is around 2.8% lower and 4.1% for the models of Resolutions CMN Nos. 3792/2009 and 4661/2018, respectively.

Considering that RV was the investment class showing the second highest volatility, this result is in line with expectations, given the greater possibility of allocation in shares without specific Corporate Governance levels in the most recent legislation (Resolution No. 4661/2018), when compared to prior regulations. However, this characteristic of exposing the EFPC portfolio to greater volatility allows the model of Resolution No. 4661/2018 to obtain a higher probability of achieving the actuarial target than the model of Resolution No. 3792/2009 when the benchmarks compared are the targets of 3%, 4%, and 5% p.y.

It is well known that, in Brazil, fixed income assets have outperformed stock indexes for more than two decades (Damasceno & Carvalho, 2021; Flores et al., 2021; Paula & Iquiapaza, 2022). The results found in this study are in line with the literature, which suggests that investors should keep a significant portion of their investments in the fixed income class, but that it may be worth carrying out a strategic allocation weighing other asset classes through indexed funds (Daltro & Leal, 2019). This is particularly important when base interest rates are at lower levels (Duijm & Bisschop, 2018), as occurred in Brazil during the pandemic.

Finally, despite the models reaching all goals, the results highlight the idea that entities still have a high probability of obtaining negative returns in their portfolios, showing that portfolios with more allocation restrictions by asset class have lower exposures, considering the results shown by the unrestricted model.

5. FINAL REMARKS

This study aimed to assess whether the new standard of thresholds established by Resolutions CMN Nos. 4661/2018 (4994/2022) was enough to achieve financial returns that cover the actuarial liabilities of EFPCs in Brazil. To do this, the behaviors of 3 different models for building portfolios were studied: (i) Unrestricted Model; and (ii-iii) Restricted Models (following the allocation thresholds established by Resolutions CMN Nos. 3792/2009, 4661/2018, and 4994/2022). Furthermore, we also provide probability measures that such models will be able to achieve varying levels of actuarial targets or generate negative returns over six time horizons (from 12 to 180 months).

Using the methodology proposed by Damasceno and Carvalho (2021), this article has as its main contribution to the literature a technical assessment of the main effects of the flexibilization of asset investment allocation thresholds in multiple investment classes proposed by Resolutions CMN Nos. 4661/2018 and 4994/2022. The relevance of this study is greater the lower the base interest rate, which is the reference remuneration for public and other RF bonds, since economic players need to take on more risks to obtain higher returns (Daltro & Leal, 2019). Also, ALM Models generally use VaR as a risk measure. However, as this measure does not satisfy the axiom of subadditivity (Artzner et al., 1999), CVaR was used, which has better properties, especially regarding the ability to diversify portfolios, precisely the object under analysis.

Thus, it was possible to verify that all actuarial targets were achieved in the simulations carried out. The unrestricted model managed to obtain higher returns and at lower volatility levels when compared to models that included legal impositions. In other words, giving in to more flexible legislation does not necessarily mean that EFPCs will incur a greater risk of losses. But it necessarily means that the guaranteeing assets of such entities will be subject to lower volatility as long as there is clearly efficient supervision by the PREVIC in a less strict scenario for asset allocation. This result shown by the unrestricted model was only possible due to the higher threshold for allocation to foreign investment assets that counterbalanced the volatility imposed by other assets (consisting mostly of national investments and which had a negative correlation with BGATT and MSCI).

It should be noticed that, in the case of making asset investment thresholds more flexible, there is a need for even greater credit and liquidity risk control, as losses arising from credit and liquidity risk (implicit in this study) could be material for an EFPC, caused by excessive concentration in specific classes.

For the sake of simplicity, in this study only the modeling of the assets end of the ALM model was considered, disregarding the biometric modeling of actuarial liabilities of DB plans. The actuarial targets were defined by having the values practiced in some of the main funds in the national market as a basis, constituting the main limitation of this paper. For further research, it is encouraged to verify the same impacts, however, also considering a liabilities ALM modeling with a wider spectrum of asset classes, as used in the open pension context by Flores et al. (2021), or adopting investments in alternative assets (e.g. commodities, mutual funds) that provide diversification benefits and are aimed at backing liabilities as suggested by Bernardo and Campani (2019).

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