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Original Article

Optimal constrained strategies for factor-based investing in the Brazilian stock market

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ABSTRACT

The paper examines investment strategies for factor-based portfolios formed by integrating a regime-switching model with a stochastic recursive utility function. Drawing from the seminal works of Fama and French (1993) and Carhart (1997), the authors identify four risk factors within the Brazilian stock market. Subsequently, employing the CGL model proposed by Campani et al. (2021), the study develops investment strategies to diversify across portfolios formed with these risk factors. The CGL model provides the framework to apply the stochastic recursive utility function to estimate the strategies based on regimes, from which the authors implement dynamic constraints to optimally control the portfolio weights. They then conduct a performance analysis through an out-of-sample exercise to compare the regime-switching strategies with benchmarks formed by single-state passive and active strategies. The empirical findings demonstrate the outperformance of regime-switching strategies, as evidenced by superior Sharpe ratios across both the complete sample and shorter subsamples within the exercise. The results also reveal that the unleveraged regime-switching strategy consistently exhibits the lowest volatility within each sample subset. In addition, the analysis of certainty equivalent returns confirms the statistical significance of the outperformance of regime-switching strategies over the benchmarks. The investigation focused on the Brazilian stock market to examine the potential benefits and efficacy of applying such a strategy in an emerging market context. Ultimately, the findings underscore that factor-based strategies formulated through a regime-switching model using a stochastic recursive utility function have the potential to outperform traditional benchmarks in terms of risk-adjusted returns within the Brazilian stock market, offering actionable insights for investors navigating the Brazilian landscape.

Keywords: regime-switching models, risk factors, stochastic differential recursive utility, dynamic asset allocation, leverage and portfolio constraints.

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Estratégias restritas ótimas para investimentos baseados em fatores no mercado de ações brasileiro

RESUMO

O artigo examina estratégias de investimento para carteiras baseadas em fatores formadas pela integração de um modelo de múltiplos regimes com uma função de utilidade recursiva estocástica. Com base nos trabalhos seminais de Fama e French (1993) e Carhart (1997), os autores identificam quatro fatores de risco no mercado acionário brasileiro. Posteriormente, empregando o modelo CGL proposto por Campani et al. (2021), o estudo desenvolve estratégias de investimento para diversificar os portfólios formados com esses fatores de risco. O modelo CGL fornece a estrutura para aplicar a função de utilidade recursiva estocástica para estimar as estratégias com base em regimes, a partir dos quais os autores implementam restrições dinâmicas para controlar de forma otimizada os pesos do portfólio. Em seguida, eles realizam uma análise de desempenho por meio de um exercício fora da amostra para comparar as estratégias de múltiplos regimes com benchmarks formados por estratégias passivas e ativas de estado único. Os resultados empíricos demonstram o desempenho superior das estratégias de múltiplos regimes, conforme evidenciado pelos índices de Sharpe superiores tanto na amostra completa quanto em subamostras mais curtas dentro do exercício. Os resultados também revelam que a estratégia de múltiplos regimes sem alavancagem apresenta consistentemente a menor volatilidade em cada subconjunto da amostra. Além disso, a análise dos retornos dos equivalentes de certeza confirma a significância estatística do desempenho superior das estratégias de múltiplos regimes em relação aos benchmarks. A investigação se concentrou no mercado de ações brasileiro para examinar os possíveis benefícios e a eficácia da aplicação dessa estratégia em um contexto de mercado emergente. Em última análise, as descobertas ressaltam que as estratégias baseadas em fatores formuladas por meio de um modelo de múltiplos regimes usando uma função de utilidade recursiva estocástica têm o potencial de superar os benchmarks tradicionais em termos de retornos ajustados ao risco no mercado acionário brasileiro, oferecendo insights práticos para os investidores que navegam no cenário brasileiro.

Palavras-chave: *modelos de múltiplos regimes, fatores de risco, utilidade recursiva diferencial estocástica, alocação dinâmica de ativos, alavancagem e restrições de portfólio.*

1. INTRODUCTION

Some critical questions for investors relate to the main drivers of stock returns and how to diversify among them. Sharpe (1964) and Lintner (1965), for instance, approached the first question by introducing the CAPM model, in which different exposures to market risk (beta) describe variations in the expected excess returns of stocks. Next, Fama and French (1993) presented a three-factor model based on the excess return of the market portfolio, the return of a portfolio long in small stocks and short in big stocks (small minus big, SMB), and the return of a portfolio long in high book-to-market stocks and short in low book-to-market stocks (high minus low, HML). They demonstrate that adding these two portfolios, often called size and value factors, as additional risk factors leads to a better explanation of the cross-section of average stock returns. Later, Carhart (1997) extended the three-factor model with a momentum factor, a portfolio long in winner stocks and short in loser stocks (momentum, MOM).

Although the literature presents additional risk factors for equities (Ang, 2014; Fama and French, 2015; Hou et al., 2015), we set our scope under the classical Fama-French-Carhart factors, i.e., the four-factor model. We follow Chincoli and Guidolin (2017), who studied the four factors as main market drivers. Thus, we will refer to SMB, HML, and MOM as the investable portfolios that mimic the size, value, and momentum factors, as in Ferson et al. (2006).

We address the second critical question concerning diversification strategies by pondering that a portfolio formed by different factors allows the investor to select between different types of risk. For instance, if one expects a factor to outperform the market, one can increase its exposure through a portfolio in which the weight of that factor is greater than that of the market. This is the transformation of passive factor investing into an active strategy, says Ang (2014).

The literature offers a rich documentation of the shortcomings of passive investing. For example, Haghani and Dewey (2016) highlight the dangers of holding a portfolio with weights predetermined by market values in the face of bubbles and market panic. Furthermore, Blitz (2020) documents that SMB and HML have individually experienced a negative average return in U.S. stocks over the period 2010-2019, just as MOM returns have declined significantly compared to the longer period 1963-2019.

On the other hand, we find support for active strategies, first, in Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), who argue that strategies that track volatility could mitigate momentum crashes. Then, Chincoli and Guidolin (2017) state that factors can help diversify beyond the traditional value-weighted approach, demonstrating the importance of estimating the conditional moments of factor returns. They show that regime-switching models provide superior estimates and more profitable portfolio strategies than other multivariate models. Similarly, Perez-Quiros and Timmermann (2000), Black and McMillan (2004), Guidolin and Timmermann (2008b), Tu (2010), and Gulen et al. (2011) show that factor-based portfolios built on regime-switching models outperform single-state benchmarks. However, none of these conclusions are based on recursive utility functions, which are considered more realistic for investment purposes. For example, Tu (2010) uses the quadratic utility function, while Chincoli and Guidolin (2017) work with the power utility function.

Recursive utility functions are ahead of these, as they assume tomorrow's felicity depends on today's felicity and are not time-additive. Moreover, they enable the configuration of investment preferences by timing the resolution of uncertainty, which impacts the allocation model by disentangling relative risk aversion from consumption decisions over time (elasticity of intertemporal substitution). The function that captures such preferences in continuous time is the stochastic differential utility function of Duffie and Epstein (1992).

The allocation strategy presented by Campani et al. (2021), i.e., the CGL model, is so far the only model using the stochastic differential recursive utility function in a regime-switching framework that provides a closed-form solution to the allocation problem. It is an approximate solution based on Campani and Garcia (2019). The authors demonstrate that it is sufficiently accurate. Before the CGL model, the literature solved dynamic allocation problems under regimes with power utility functions using numerical methods, such as Monte Carlo simulation, as in Sass and Haussmann (2004), Guidolin and Timmermann (2007), and Liu (2011), or under very specific conditions, as in Wachter (2002), who provides analytical formulas for solving power utility problems, but limited to a perfect negative correlation between the asset returns and the predictor variable; and Honda (2003), who finds a closed-form solution to the portfolio problem limited to the case of a constant relative risk aversion equal to 0.5.

Rouwenhorst (1999) presents evidence that emerging market stocks, like those from developed markets, exhibit SMB, HML, and MOM. He conducts an international study with 1,705 stocks from 20 emerging markets from all continents. In the Americas, for instance, he studies firms from Argentina, Brazil, Chile, Colombia, Mexico, and Venezuela. Without proposing an allocation strategy, his research shows that the factors that drive cross-sectional differences in expected stock returns in emerging equity markets are qualitatively similar to those in developed markets. In turn, Chague and Bueno (2008) and Santos et al. (2012) reveal that the three- and four-factor models are valid for Brazilian stocks, respectively. However, both applied passive allocation strategies. Meanwhile, Chen and Kawaguchi (2018) study a factor-based portfolio under a regime framework, using the quadratic utility to form strategies with Chinese stocks. Following the gap related to developed markets, to our knowledge, there are no applications of a factor-based portfolio in emerging markets using an optimal allocation with recursive utility configured under hidden regimes in a Markov process.

The CGL model has never been applied from the perspective of factor-based investing, i.e., investors who form their portfolios using risk-factor portfolios or indices. Lewin and Campani (2020a) demonstrate the CGL model's accuracy with a portfolio of Brazilian equities, bonds, and international stocks impacted by the exchange rate. In an out-of-sample exercise, Lewin and Campani (2020b) show that the CGL model outperforms benchmark portfolios of equities, different bond classes, and the exchange rate. Meanwhile, Lewin and Campani (2022) apply the CGL model in the US stock market using different levels of transaction cost. The latter authors also introduce a leverage control to allow constrained strategies in the CGL model framework, which affects regime-based portfolios, and will be addressed in the current study.

We investigate the performance of a factor-based strategy formulated with a regime-switching model using this state-of-the-art function, the recursive utility function. Our regional choice was Brazil due to its representativeness among other emerging stock markets. Furthermore, it allowed us to focus on a single country, avoiding the direct impact of exchange rates over the regime estimation.

This study computes SMB, HML, and MOM portfolios from the B3 Brazilian Stock Exchange, then applies the constrained CGL model to estimate the strategies based on regimes for the factor-based investor. Finally, in an out-of-sample exercise with transaction costs, we compare the CGL performance to active and passive strategies interpreted as benchmarks. The results indicate that the Sharpe ratio from the CGL model outperforms the benchmarks, and the certainty equivalent returns show that this outperformance is statistically significant.

2. METHODOLOGY

2.1 The Regime-Switching Economy

We consider an investor in a continuous-time model with a regime-switching economy governing asset returns. The investor maximizes her stochastic differential recursive utility function with an optimal portfolio allocation strategy.

2.1.1 State variable

Following Hamilton (1989), we consider an economy governed by the unobservable state variable Y_t describing an independent Markov chain process, given $R = \{1,2, \ldots, m\}$, where R is a finite set of m possible regimes. Then, we treat the behavior of the state variable by transition probabilities, which will determine whether the economy remains in the same regime or jumps to a new one after an exponentially distributed length of time, as follows:

$$P_{ij,\Delta t} = \frac{\lambda_{ij}}{\sum_{k\neq i}\lambda_{ik}}\left(1 - e^{-\sum_{k\neq i}\lambda_{ik}\Delta t}\right), \quad \text{with } j \neq i \in R \text{ and } \lambda_{ij} > 0, \tag{1}$$

where $\lambda_{ii} = -\Sigma_{k\neq i}\lambda_{ik} \leq 0$ such that $P_{ij,\Delta t} = \frac{\lambda_{ij}}{-\lambda_{ii}}(1 - e^{\lambda_{ii}\Delta t})$. The probability of staying in the same regime i over the next Δt is given by $P_{ii,\Delta t} = e^{\lambda_{ii}\Delta t}$, where λ_{ij} is assumed to be constant.

2.1.2 Asset dynamics

The risk factors are represented by n risky assets, and the excess returns ($\hat{r}$) over the riskless asset (rf) are defined through the multidimensional stochastic process:

$$\begin{bmatrix} d\hat{r}_{1,t} \\ d\hat{r}_{2,t} \\ \ldots \\ d\hat{r}_{n,t} \end{bmatrix} = \boldsymbol{\mu}_{s,t}\, dt + \boldsymbol{\sigma}_{s,t}\, \boldsymbol{dZ}_t = \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \\ \ldots \\ \mu_{n,t} \end{bmatrix} dt + \begin{bmatrix} \sigma_{11,t} & 0 & \ldots & 0 \\ \sigma_{21,t} & \sigma_{22,t} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_{n1,t} & \sigma_{n2,t} & \ldots & \sigma_{nn,t} \end{bmatrix} \begin{bmatrix} dZ_{1,t} \\ dZ_{2,t} \\ \ldots \\ dZ_{n,t} \end{bmatrix}, \tag{2}$$

where $\boldsymbol{\mu}_{s,t}$ is an $n \times 1$ vector of the instantaneous expected risk premia (drifts), $\boldsymbol{\sigma}_{s,t}$ is an $n \times n$ lower triangular volatility matrix, and $\boldsymbol{dZ}_t$ is a column vector with n increments of independent standard Wiener processes. Given that both $\boldsymbol{\mu}_{s,t}$ and $\boldsymbol{\sigma}_{s,t}$ are time-varying and conditioned by the state variable Y_t, if we consider $Y_t = i, i \in R$, we obtain:

$$\boldsymbol{\mu}_{s,t} = \boldsymbol{\mu}_{s,i} = \begin{bmatrix} \boldsymbol{\mu}_{1,i} \\ \boldsymbol{\mu}_{2,i} \\ \ldots \\ \boldsymbol{\mu}_{n,i} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\sigma}_{s,t} = \boldsymbol{\sigma}_{s,i} = \begin{bmatrix} \sigma_{11,i} & 0 & \ldots & 0 \\ \sigma_{21,i} & \sigma_{22,i} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_{n1,i} & \sigma_{n2,i} & \ldots & \sigma_{nn,i} \end{bmatrix}, \tag{3}$$

where $\boldsymbol{\mu}_{j,i}$ coefficients and) $\boldsymbol{\sigma}_{s,i}$ matrices are constant for each $j = \{1,2, \ldots, n\}$. We emphasize that $\boldsymbol{\sigma}_{s,i}$ elements are defined as partial volatilities, e.g., $\sigma_{21,i}$ denotes the partial volatility of asset 2 in relation to the first Wiener process ($dZ_{1,t}$) in regime i, and also that $\boldsymbol{\sigma}_{s,i}\boldsymbol{\sigma}^{T}_{s,i}$ represents the regime-dependent variance-covariance matrix. As the drifts are regime-dependent and simultaneously time-dependent, it means that they can vary in time even if the regime remains unchanged. Such drifts are stored in an $n \times m$ drift matrix:

$$\boldsymbol{D}_{s,t} = \begin{bmatrix} \boldsymbol{\mu}_{1,1} & \boldsymbol{\mu}_{1,2} & \ldots & \boldsymbol{\mu}_{1,m} \\ \boldsymbol{\mu}_{2,1} & \boldsymbol{\mu}_{2,2} & \ldots & \boldsymbol{\mu}_{2,m} \\ \ldots & \ldots & \ldots & \ldots \\ \boldsymbol{\mu}_{n,1} & \boldsymbol{\mu}_{n,2} & \ldots & \boldsymbol{\mu}_{n,m} \end{bmatrix}. \tag{4}$$

With such assets and state variable processes, we obtain the regime parameters through the maximum likelihood (ML) estimation methodology, which are: the drift matrix $\boldsymbol{D}_{s,t}$, the volatility matrix $\boldsymbol{\sigma}_{s,i}$ and the transition probabilities $P_{ij,\Delta t}$ (where $j \neq i \in R$). The number of parameters to be estimated is $[mn + mn(n + 1)] \div 2 + m(m - 1)]$. Given the unobservable nature of the regimes, following Hamilton (1989), we assume that investors can infer the occurrence of the regimes through filtered probabilities by observing the past returns of the assets.

2.2 The Portfolio Strategy

Considering W_t as the wealth in t and $\boldsymbol{\alpha}_t$ as the $1 \times n$ vector of portfolio shares of the risky assets, and $(1 - \boldsymbol{\alpha}_{t1})$ as the riskless asset share, the wealth dynamics can be expressed as:

$$dW_t = (1 - \boldsymbol{\alpha_t} \mathbf{1}) W_t r_f \, dt + W_t \boldsymbol{\alpha_t} \frac{dS_t}{S_t} = W_t r_f \, dt + W_t \boldsymbol{\alpha_t} [\boldsymbol{D_{s,t}} \boldsymbol{\pi_t} \, dt + (\boldsymbol{V\pi_t}) \, \boldsymbol{dZ_t}], \quad (5)$$

where $\mathbf{1}$ is a column vector of n ones, $\frac{dS_t}{S_t}$ is the column vector with n infinitesimally risky asset returns, $\boldsymbol{\pi}_t$ is a column vector of the m filtered probabilities in t, and $\boldsymbol{V}$ is an $1 \times m$ row vector containing the regime-dependent covariance matrices $(\boldsymbol{\sigma}_{s,i})$.

2.2.1 Utility function

In the CGL model, the investor's preferences are characterized as continuous-time and modeled by the stochastic utility function of Duffie and Epstein (1992):

$$J_t = E_t \left[\int_{u=t}^{T} f(C_u, J_u) \, du + \frac{W_T^{1-\gamma}}{1-\gamma} \right], \quad (6)$$

where E_t is the expected value at the current moment (t); T is the investment horizon; f is the recursive aggregator of the utility function J_t as a function of the consumption rate C_u (at moment u) and J_u, the continued utility at u. In turn, W_T is the investor's terminal wealth, while γ is the risk aversion coefficient. The following function details the utility function aggregator:

$$f(C, J) = \frac{\beta}{1 - \frac{1}{\psi}} (1-\gamma) J \left\{ \left[\frac{C}{[(1-\gamma) J]^{\frac{1}{1-\gamma}}} \right]^{1-\frac{1}{\psi}} - 1 \right\}, \quad (7)$$

where β is the time preference rate of the investor's utility (felicity) and ψ is the elasticity of intertemporal substitution, i.e., consumption choices over time. Thus, we must set ψ, β and γ to configure the strategy using the recursive utility.

Campani and Garcia (2019) analyze the sensitivity of consumption and portfolio choices in a single-state model. They find that ψ affects consumption preferences but barely affects the allocation strategy. Campani et al. (2021) find a similar result for a regime-switching model. So, we can study the allocation strategy and disregard intermediary consumption by defining $\psi = \infty$. This represents the investor waiting for the final horizon to consume the wealth. In a problem without intermediary consumption, the value of β will not significantly affect the allocation strategy. Campani et al. (2021) and Guidolin and Timmermann (2007) also show that the investment horizon has a negligible impact on the strategy when considering frequent rebalancing. And like both, we consider $\gamma = 5$.

Campani et al. (2021) show that the general solution quantifying the investor's total optimal utility in t ($V_t = \sup J_t$) admits the wealth-separable solution:

$$V(W_t, \boldsymbol{\pi_t}, \tau) = H(\boldsymbol{\pi_t}, \tau) \frac{W_t^{1-\gamma}}{1-\gamma}, \quad (8)$$

where $\tau = T - t$ is the time until the final horizon and $H(\boldsymbol{\pi}_t, \tau)$ is a function in terms of the time to the horizon and the regime probability vector. However, as an exact analytical expression for $H(\boldsymbol{\pi}_t, \tau)$ is not yet available in the literature, Campani et al. (2021), based on the Bellman equation, solve the problem with the following approximate analytical expression:

$$H(\boldsymbol{\pi_t}, \tau) = exp\left[A_0(\tau) + \textstyle\sum_{i=1}^{m} A_i(\tau) \, \pi_{i,t} + \sum_{i=1}^{m} B_i(\tau) \pi_{i,t}^2 + \sum_{j<i} C_{ij}(\tau) \, \pi_{i,t} \, \pi_{j,t} \right], \quad (9)$$

where $\pi_{i,t}$ is the probability of regime i at time t. Meanwhile, A_0, A_i, B_i, and C_{ij} are time-horizon coefficients obtained from solving the Bellman equation under a system of partial differential equations (PDE). Campani et al. (2021) demonstrate the PDE and provide the details for solving the Bellman equation.

2.2.2 Portfolio weights

Given the approximate solution for $V(W_t, \boldsymbol{\pi}_t, \tau)$ and the coefficients A_0, A_i, B_i, and C_{ij} from the function $H(\boldsymbol{\pi}_t, \tau)$, the CGL model presents the optimal weights for the regime-switching allocation using the recursive utility function given by the following form:

$$\boldsymbol{\alpha_t} = \tfrac{1}{\gamma} \left(\boldsymbol{D_{s,t}} \boldsymbol{\pi_t} \right)^T [(\boldsymbol{V\pi_t})(\boldsymbol{V\pi_t})^T]^{-1} + \tfrac{1}{\gamma} \textstyle\sum_{i=1}^{m} \left[A_i(\tau) + 2B_i(\tau) \, \pi_{i,t} + \sum_{j \neq i} C_{ij}(\tau) \, \pi_{j,t} \right] \boldsymbol{\sigma_{i,\pi}} \, (\boldsymbol{V\pi_t})^{-1}, \quad (10)$$

where $\boldsymbol{\alpha}_t = [\boldsymbol{\alpha}_{1,t} \dots \boldsymbol{\alpha}_{n,t}]$, $\boldsymbol{\sigma}_{i,\pi} = [\sigma_{i1,\pi} \, \sigma_{i2,\pi} \dots \sigma_{in,\pi}]$, $i \in R$, and $j = \{1, 2, \dots, n\}$.

2.2.3 Maximum leverage control (MaxLev)

Equation 10 indicates the optimal weights, but we must control them within the maximum leverage permitted by the investor. Hence, we apply Lewin and Campani's (2022) MaxLev to constrain leverage in the CGL model. First, we collect the unconstrained weights at 100% probability for each regime through equation (10) with $\boldsymbol{\hat{\pi}_i} = i^{th}$ column of an m-order identity matrix. Then, using a $1 \times m$ vector, we store the unconstrained leverages conditioned by each regime (***UncLev***), whose elements are:

$$UncLev_i = \left[\sum_{k=1}^{n+1} \max\left(\boldsymbol{\hat{\alpha}}_{\boldsymbol{\hat{\pi}_i,k}}, 0\right)\right] - 1, \text{ with } i \in R = \{1, 2, \dots, m\}, \quad (11)$$

where $\boldsymbol{\hat{\alpha}}_{\boldsymbol{\hat{\pi}_i,k}}$ is the k^{th} element of the row vector $\boldsymbol{\hat{\alpha}}_{\boldsymbol{\hat{\pi}_i}} = [\boldsymbol{\alpha}_{\boldsymbol{\hat{\pi}_i}} \quad (1 - \boldsymbol{\alpha}_{\boldsymbol{\hat{\pi}_i}}\mathbf{1})]$ formed by the risky asset weight vector and the riskless asset weight, all conditioned by $\boldsymbol{\hat{\pi}_i}$. We define the regime conditioned maximum leverage values ($L_{b,i}$) by building a $z \times m$ matrix, where z is a positive integer representing the number of policies investigated in the research and $b = \{1,2, \dots, z\}$:

$$\boldsymbol{MaxLev} = \begin{bmatrix} L_{11} & \cdots & L_{1m} \\ \vdots & \ddots & \vdots \\ L_{z1} & \cdots & L_{zm} \end{bmatrix}. \quad (12)$$

Then, we compute the necessary adjustments for γ to confine $UncLev_i$ within the limits given in ***MaxLev***, with which we derive the risk parameter ($\gamma = 5$) conditional on the limits and regimes. The procedure emerges on a new $z \times m$ matrix whose elements are:

$$\gamma_{b,i} = \gamma \times max\,[\,(1 + UncLev_i) \div (1 + L_{b,i})\,, 1\,], \quad (13)$$

where the maximum operator preserves $\gamma = 5$ when the element $UncLev_i$ is already below the limit imposed by $L_{b,i}$ Multiplying the new matrix b^{th} row (expressed by $\boldsymbol{\gamma}_b = [\gamma_{b,1} \dots \gamma_{b,m}]$) by the column vector of regime probabilities at time t ($\boldsymbol{\pi}_t$), we obtain a dynamic value as our risk parameter, which is an average of $\boldsymbol{\gamma}_b$ (dynamically) weighted by $\boldsymbol{\pi}_t$. Plugging this into equation (10), we then find the weights constrained by policy b at time t:

$$\boldsymbol{\alpha_{b,t}} = \frac{1}{(\boldsymbol{\gamma_b \pi_t})} \left(\boldsymbol{D_{s,t}\, \pi_t}\right)^T [(\boldsymbol{V\pi_t})(\boldsymbol{V\pi_t})^T]^{-1} + \frac{1}{\gamma}\sum_{i=1}^{m}\left[A_i(\tau) + 2B_i(\tau)\,\pi_{i,t} + \sum_{j\neq i} C_{ij}(\tau)\,\pi_{j,t}\right] \boldsymbol{\sigma_{i,\pi}}\,(\boldsymbol{V\pi_t})^{-1}. \quad (14)$$

2.3 The Four-Factor Model

2.3.1 Data set

We calculated the risk factors using the stocks listed in the IBrX 100, the index of the 100 most liquid Brazilian stocks. The B3 exchange rebalances the index composition on the first Monday of January, May, and September. This database starts in 1996. We excluded the three compositions from the initial year due to their higher number of ineligible stocks compared to those from 1997 onward, as per the selection criteria specified below. We extracted the daily prices from 1997 to 2022 from Economatica (we converted the series to weekly data to apply the regime-switching model).

Based on rebalancing dates, we update the stock universe three times a year to keep up with new index compositions. In addition, we control for survival bias by considering the complete set of stocks listed in the index composition at any given time. Thus, when necessary, we calculated the portfolios by including stocks that were subsequently delisted. Using the index naturally imposed a liquidity filter on the stock selection. In addition, first, we kept only one stock per company (the most traded). Second, we filtered out penny stocks, stocks with a negative book value (to eliminate those with a high default risk), and stocks without trades in more than 20% of the daily observations in the year before the composition report (to mitigate potential distortions in the MOM calculation).

2.3.2 Construction

Following Fama and French (1993) and Cahart (1997), the four-factor model relies on multivariate sorting to build value-weighted portfolios based on size (market capitalization), book-to-market (BM), and prior (2-12) returns (since the first factor is the market portfolio). We followed Fama (2017) to set the prior (2-12) configuration. This means, for example, that in January 2001, stocks were ranked according to prior 2-12 month total returns based on their continuously compounded returns from January 2000 to November 2000.

The cited authors create six bivariate portfolios. They classify the stocks into two portfolios ranked by size, using the median to separate big and small stocks. At the same time, they classify the stocks into three portfolios ranked by BM, using the 30th and 70th percentiles to separate high (value), neutral, and low (growth) stocks. Applying the same procedure using past returns instead of BM, they also get high (winners), neutral, and low (losers) stocks. By combining the sort by size with the sort by BM, they obtain six bivariate portfolios. Similarly, by combining the sorts by size and prior returns, they obtain another six portfolios.

Although we also build bivariate portfolios, we construct nine portfolios instead of six. First, we classify the stocks into three portfolios ranked by size, using the 40th and 60th percentiles to separate them into big, neutral (medium), and small stocks; we adopt this same procedure to rank the stocks by BM and prior returns. The configuration with these percentiles means that we shrank the neutral (univariate) portfolios relative to Fama and French (1993); the reason is that we have far fewer stocks in Brazil, so we want to omit fewer stocks (as we explain later, the neutral portfolios are disregarded). Next, by combining the sort by size with the sort by BM, we obtain nine bivariate portfolios. Then, we obtain another set of nine portfolios by repeating the procedure using prior returns instead of BM. Note that five out of each set of nine portfolios are bivariate types, where "neutral" is at least one of the possible combinations.

From the nine possible combinations formed between size and BM (or between size and prior returns), we discard the five portfolios formed with "neutral" to mitigate the correlation between factors. Finally, we repeat the sorting of the portfolios on a weekly basis to match the data frequency used by the CGL model.

2.3.3 SMB (small minus big)

As indicated, when sorting size and BM, after discarding the five portfolios formed with "neutral," we obtain four portfolios: small value, small growth, big value, and big growth. Then, using the daily returns of these portfolios, we obtain SMB as:

$$SMB = \frac{(Small\,Value + Small\,Growth)}{2} - \frac{(Big\,Value + Big\,Growth)}{2}. \quad (15)$$

2.3.4 HML (high minus low)

The abovementioned portfolios, following Fama (2017), provide HML as:

$$HML = \frac{(Small\,Value + Big\,Value)}{2} - \frac{(Small\,Growth + Big\,Growth)}{2}. \quad (16)$$

2.3.5 MOM (momentum)

The nine bivariate portfolios formed by size and prior (2-12) returns give rise to MOM, where, analogous to the previous factors, we discard the five portfolios formed by the combinations with "neutral." Also following Fama (2017), MOM is defined as follows:

$$MOM = \frac{(Small\,High + Big\,High)}{2} - \frac{(Small\,Low + Big\,Low)}{2}, \quad (17)$$

where small high, big high, small low, and big low are the series of daily returns generated by the bivariate portfolios sorted by size and prior (2-12) returns.

2.3.6 Market factor (Mkt-*rf*)

The excess returns of the IBrX 100 index over the returns of the risk-free asset represent the market factor. Following Lewin and Campani (2020a), we consider the return of the Brazilian CDI as the risk-free asset (*rf*).

2.4 Application of the CGL Model

We apply the CGL model to allocate $n = 4$ risky assets (Mkt-*rf*, SMB, HML, and MOM) along with the risk-free asset (CDI).

2.4.1 Out-of-sample exercise

We constructed a 20-year exercise organized in 60 observation windows. All of them started on January 8th, 1997, but the number of observations was increased in 4-month steps; this procedure guarantees that we estimate the model based on the richest possible sample. We (re) estimate the model every four months, at the end of each window (a higher frequency did not significantly change the estimations). The first window, ending on December 30th, 2002, has 313 observations; the last one, ending on August 31st, 2022, has 1339 observations. Hence, the regime parameters were (re)estimated via maximum likelihood with the observations of each window and held over the following four months. At every new week, we define the strategy from the filtered probabilities estimated in t for $t + 1$. Then, replicating only the information available at the time of the investment decision, we observe the out-of-sample returns. This exercise extends from January 8th, 2003 to December 19th, 2022, encompassing 1356 weekly observations.

Table 1
Information criteria

Window	m	AIC	BIC	H-Q
Oldest				
	2	-5.039	-5.024	-5.075
	3	-5.067	-5.043	-5.124
	4	-5.085	-5.051	-5.167
Intermediate				
	2	-14.854	-14.826	-14.886
	3	-14.923	-14.878	-14.974
	4	-15.033	-14.970	-15.106
Most recent				
	2	-24.435	-24.401	-24.465
	3	-24.616	-24.562	-24.665
	4	-24.809	-24.732	-24.877

Notes: *The table presents the information criteria for the models with n = 4 risky assets under 2,3, and 4 regimes. Its columns show Akaike (AIC), Bayes-Schwartz (BIC), and Hannan-Quinn (H-Q) for three windows from the out-of-sample exercise. The oldest window was estimated from Jan/08/1997 to Dec/30/2002, the intermediate window from Jan/08/1997 to Dec/26/2012, and the most recent window from Jan/08/1997 to Dec/19/2022. The entire out-of-sample exercise was performed with 60 windows.*
Source: *Prepared by the authors.*

2.4.2 Number of regimes

To define the number of regimes (m), we consider $m = 1$ as the single regime model presented in the Results section and that $m = 4$ is the highest number of states often observed in the literature. In addition, Guidolin and Ono (2006) indicate a saturation ratio between the number of estimated parameters and the series length, with values above 17. In our application, it is $m \leq 4$. Thus, we tested models with $m = 2, 3, 4$. Table 1 shows their information criteria (IC), presenting 3 out of the 60 (re)estimations obtained using windows 1,30, and 60. The IC is relatively stable during the (re)estimations and, as Table 1 indicates, $m = 4$ dominates the other models. Therefore, we apply a model with four regimes.

2.4.3 Transaction costs

We assume that the investor rebalances the portfolio every (end-of-) week and incurs transaction costs. We follow Gârleanu and Pedersen (2013) and Nystrup et al. (2019), who set transaction costs at 10 basis points (0.10%) for dynamic asset allocation strategies. The latter authors additionally propose to include holding costs, charged at the risk-free rate over the short sales. In our application, holding costs are naturally considered, since the investor borrows at the risk-free rate for short selling.

2.5 Portfolio Leverage

We apply Lewin and Campani's (2022) MaxLev procedure to constrain the optimal CGL solution, to present the portfolios according to leverage levels, and to observe different investment profiles. The unleveraged strategy is CGL MaxLev 0%, and the leveraged ones are CGL MaxLev 50%, 100%, 150%, 200%, and 250%. Section 4.3 shows the CGL results obtained using these configurations and identifies those closest to the benchmarks' leverage in order to perform the exercise comparisons appropriately.

Nonetheless, equations 15 to 17 show that factors have intrinsic leverage as they long and short their underlying portfolios, generating the risk premia over the risk-free asset (CDI). Equation 10 presents the vector of optimal risky asset weights that creates the factor-based strategy. It is also the vector of the optimal weights for each factor, which do not add up to 100%, as it has no risk-free weight $(1 - \boldsymbol{\alpha}_t \mathbf{1})$. To compute the dynamically constrained weights (equation 14), we assume that the weights sum to 100% by considering that a \$100 investment in SMB, for example, is investing \$100 in the risk-free asset, buying \$100 of small stocks (long), and short selling \$100 in the big stocks portfolio.

Therefore, an unleveraged strategy, i.e., CGL MaxLev 0%, indicates that we do not suggest any extra leverage besides the intrinsic leverage of the factors (100%). In turn, if the CGL model recommends 50% leverage, the investor will be 150% leveraged, given the intrinsic leverage of the factors.

2.6 Benchmarks

We assume that the investor is willing to diversify between the Brazilian risk factors. Thus, we compare the CGL model strategies with four active and passive strategies. The benchmarks also differentiate between unleveraged and leveraged strategies, as we will present the CGL portfolio varying the leverage configuration.

2.6.1 Equal-weighted portfolio (1/*n*)

DeMiguel et al. (2009) demonstrate that a $1/n$ portfolio outperforms several dynamic models based on optimal rules, despite being a naïve strategy. Thus, it represents a benchmark for passive (and unleveraged) strategies.

2.6.2 Tangency portfolio (Tangency)

The tangency portfolio is based on the quadratic utility preferences used to build the efficient frontier. The point where the upward-sloping line is tangent to the frontier of risky assets corresponds to this portfolio, a portfolio of risky assets only. We present it as a benchmark for the active unleveraged strategy, i.e., CGL MaxLev 0%, as it maximizes the Sharpe ratio (over the risky portfolios on the efficient frontier).

2.6.3 Single regime model (SR)

The SR model corresponds to the recursive utility preferences of an investor who does not consider a multi-regime economy. We use it to assess the impact of regime switching on overall performance. As the SR strategy relies on leverage, it is a benchmark for the active leveraged strategies, i.e., CGL MaxLev 50% to 200%.

2.6.4 Constrained single regime model (SR CONS)

First, we constrain the latter strategy by recalculating the optimal weights proportionally to an unleveraged portfolio. Then, SR CONS becomes a benchmark for active unleveraged strategies against which we assess the impact of regime switching on CGL MaxLev 0%.

2.7 Robustness Check

Following Fugazza et al. (2015) and Campani et al. (2021), we use the annualized certainty equivalent returns (*CER*) to compare and rank different strategies. The authors provide the derivation of the following expression, which is used to compute *CER*:

$$CER_i(\gamma,t) \equiv \frac{F}{T}\left\{\frac{1}{W_t}\left[\frac{1}{K-T}\sum_{\tau=1}^{K-T}\left[W_{\tau+T}\left(\widehat{\omega}_{i,t}(\gamma,T)\right)\right]^{\frac{1}{1-\gamma}}\right]-1\right\}, \quad (18)$$

where F is the data frequency (52 weeks per year), T is the horizon (520 weeks), K is the number of out-of-sample returns, $\widehat{\omega}_{i,t}$ are the proportions of the wealth invested in asset i, and W_t is the initial wealth (set to 1). The following section presents the *CER* differences to compare two portfolios. It also reports the 95% bootstrapped confidence intervals drawn from 1,000,000 samples with replacement, using the bias-corrected and accelerated percentile method due to non-normalities in the out-of-sample returns.

3. RESULTS

3.1 The Four-Factor Model

In Table 2, panel A presents the long-run mean, volatility, and correlation of the factors as single-state parameters. It shows that the expected excess returns of MOM are positive over the period 2003-2022, while those of SMB and HML are not far from zero, indicating that big and growth stocks (low BM) did not outperform the short selling of their peers, as winners did. As in Fama and French (1993), we expected to see a negative correlation between SMB and HML, as they are derived from size and BM portfolios. Given that BM is a ratio of book equity (BE) to market equity (size), and size is updated daily (as a result of price), but BE changes at a lower frequency, in the long run, HML usually has a constant numerator and a denominator that varies with size. Thus, the correlations from HML and SMB with Mkt-*rf* also present inverted signs. In the next section, reading the regime parameters, we derive the intuition behind this.

Table 2
Estimated parameters

Panel A: Single Regime Model			Mkt-rf	SMB	HML	MOM
Expected returns						
			4.2%	-0.4%	0.0%	9.3%
Volatility and correlation matrix						
		Mkt-*rf*	27.9%			
		SMB	-0.30	20.2%		
		HML	0.30	-0.42	18.0%	
		MOM	-0.18	-0.01	-0.32	19.0%
Panel B1: Four State Model			**Mkt-*rf***	**SMB**	**HML**	**MOM**
Expected returns	Regime 1 (crash)		-64.8%	-41.8%	-43.7%	61.3%
	Regime 2 (bear market)		-6.7%	5.3%	14.6%	-4.3%
	Regime 3 (bull market)		2.9%	-4.6%	-3.0%	9.6%
	Regime 4 (rally)		23.2%	3.8%	-2.1%	14.6%
Volatility and correlation matrix						
	Regime 1 (crash)	Mkt-*rf*	86.6%			
		SMB	-0.59	63.9%		
		HML	0.57	-0.82	37.4%	
		MOM	-0.15	0.39	-0.49	36.2%
	Regime 2 (bear market)	Mkt-*rf*	29.5%			
		SMB	-0.12	22.4%		
		HML	0.03	-0.41	21.4%	
		MOM	-0.09	-0.32	0.01	22.2%
	Regime 3 (bull market)	Mkt-*rf*	21.2%			
		SMB	-0.12	14.7%		
		HML	0.68	-0.09	18.1%	
		MOM	-0.66	0.12	-0.75	22.3%
	Regime 4 (rally)	Mkt-*rf*	17.6%			
		SMB	-0.29	11.9%		
		HML	0.16	-0.35	12.0%	
		MOM	0.12	0.09	-0.29	11.4%
Panel B2: Four State Model			**Regime 1**	**Regime 2**	**Regime 3**	**Regime 4**
Transition probabilities						
	Regime 1 (crash)		86.9%	12.9%	0.1%	0.1%
	Regime 2 (bear market)		1.6%	87.3%	1.1%	10.0%
	Regime 3 (bull market)		0.0%	0.0%	96.7%	3.3%
	Regime 4 (rally)		0.0%	7.0%	1.0%	92.0%
Ergodic probabilities			3.6%	28.7%	22.8%	45.0%
Duration (weeks)			8	8	30	12

Notes: *The table was computed using excess returns over the risk-free rate. Correlation matrices show the volatilities in their diagonals. Weekly returns and volatilities are annualized for presentation. The parameters were estimated using the complete data set. Market and rf returns are the IBrX 100 and the CDI rate returns, respectively. SMB, HML, and MOM are small minus big, high minus low, and momentum, respectively.*
Source: *Prepared by the authors.*

3.2 The Four-Regime Model

In Table 2, panels B1 and B2 show the parameters of the four economic states. The crash is the most pessimistic regime for Mkt-*rf*, SMB, and HML returns, and their correlation is higher than in other regimes, suggesting that size and BM have a limited impact on crash returns. In contrast, the crash is the most optimistic regime for MOM: the estimated parameters show that crash and rally states generate momentum for stocks. That is, when the market generates either positive or negative extreme risk premia, the distance between winners and losers (high and low momentum) widens. On the other hand, such an effect is not observed within the book-to-market ratios, which we can assume is due to the updating frequency of the book value, which is often longer than the regime duration. Nevertheless, the ergodic probabilities and duration still reveal that crashes are rare and short, occurring only 3.58% of the time and lasting on average 8 weeks. Most of the time, the economy is in one of the three remaining states. Bear and bull markets are the least extreme regimes, but both usually transition to a rally, the most likely of which occurs 44.95% of the time.

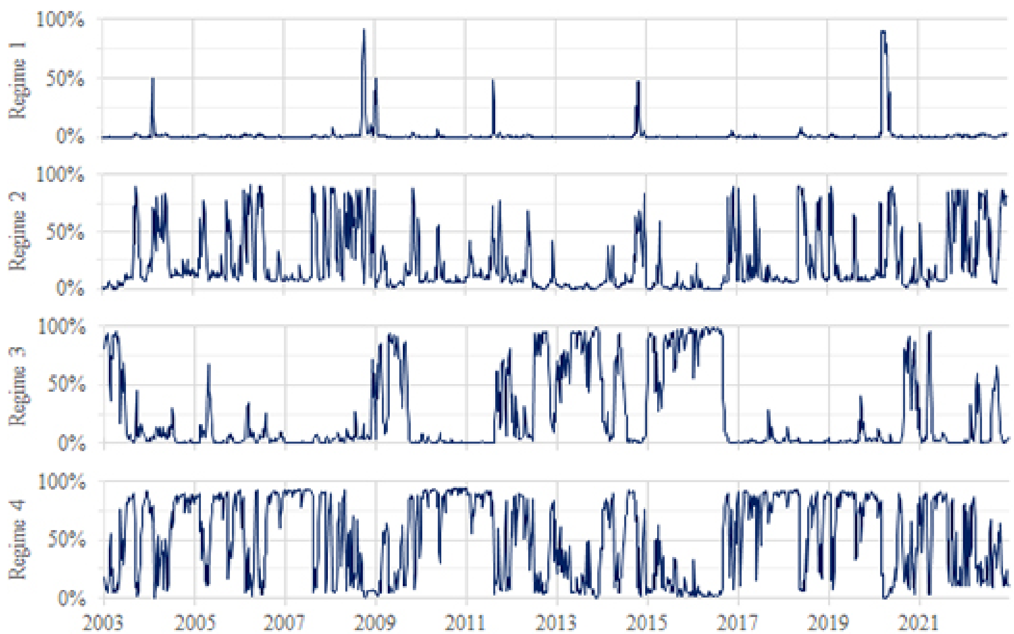

Figure 1 *Out-of-sample probabilities*
Notes: *The out-of-sample probabilities are the filtered probabilities from t + 1 in t, estimated considering the data windows described in Section 2.4.*
Source: *Prepared by the authors.*

Figure 1 shows that the bull market, despite its long duration, is less recurrent than the bear market and the rally state. The last five years of the exercise illustrate the relevance of a regime-switching model for an active strategy in a factor-based portfolio. Despite the 2020 crash and the 2021 bull market, since 2017, the most frequent regimes are the bear market and rally. Although the expected returns of SMB are roughly similar in the bear market and rally, they are not as impactful as those of HML and MOM in each state. The returns of Mkt-*rf*, HML, and MOM change signs between these two states, and a passive or active investor who does not consider the regime-switching framework would likely not capture such state shifts.

Table 3
Out-of-sample results

Strategies	Return (%)	Vol. (%)	Skewness (%)	Kurtosis	MDD (%)	Leverage (%)		
						5th Perc.	Average	Maximum
Unleveraged single-state models								
1/*n*	14.58	8.06	-108.96	7.02	16.06	0.00	0.00	0.00
SR CONS	14.80	9.99	-75.48	5.59	16.87	0.00	0.00	0.00
Leveraged single-state models								
Tangency	15.05	13.03	-106.38	9.70	32.06	0.00	6.08	55.33
SR	14.54	11.17	-80.34	4.54	20.22	0.00	12.51	37.76
Unleveraged regime-switching model								
CGL MaxLev 0%	15.65	6.33	-19.06	2.18	5.97	0.00	0.00	0.00
Leveraged regime-switching models								
CGL MaxLev 50%	17.76	10.55	-22.67	1.72	13.12	0.00	29.97	50.00
CGL MaxLev 100%	19.76	14.81	-18.97	2.30	19.25	4.59	65.11	100.00
CGL MaxLev 150%	21.56	18.74	-14.59	2.85	28.73	9.79	99.28	150.00
CGL MaxLev 200%	23.16	22.24	-20.14	2.96	37.84	15.46	131.71	200.00
CGL MaxLev 250%	24.28	25.25	-31.39	3.15	45.22	17.96	160.12	250.00
CGL Unconstrained	28.86	34.40	-37.99	4.52	55.05	25.40	244.02	716.84

Notes: *The table presents the results of the out-of-sample exercise from Jan/8/2003 to Dec/19/2022. It shows annualized weekly average returns and volatilities (vol.). Returns are absolute, i.e., not excess returns, and net of transaction costs. The table also shows skewness, kurtosis (not excess kurtosis), and maximum drawdown (MDD). Leverage is the sum of all strategy weights exceeding 100% for each observation; it does not take into account the intrinsic leverage of the factors. The right-most columns show the 5th percentile, average, and maximum leverage. The minimum leverage is zero for the optimal CGL model; therefore, it is also zero for each CGL variation. CGL MaxLev 0% to 250% use the leverage constraining method introduced by Lewin and Campani (2022), while CGL unconstrained is the strategy without it. The SR and CGL strategies were optimized for γ = 5.*
Source: *Prepared by the authors.*

3.3 Out-of-Sample Performance

Table 3 presents the factor-based portfolios using the single-state and regime-switching strategies. The 1/*n* equally weighted portfolio is the passive strategy benchmark. Its volatility is lower than any other single-state active strategy in the exercise, but higher than CGL MaxLev 0%, whose returns outperform those of the single state strategy. The higher-order statistical moments (skewness and kurtosis) also suggest that the CGL model offers lower risk than the benchmarks. At the same time, CGL MaxLev 0% had the only single-digit maximum drawdown (MDD). It reveals the promising results of using regime-switching models to diversify factor-based portfolios by employing a more realistic utility function, the stochastic recursive utility function.

Table 3 shows that the leverage of the single-state models ranges up to 55.33%, making CGL MaxLev 50% their most suitable benchmark. Moreover, their 5th percentile and average leverage are the same or the closest, while any maximum leverage set between 0% and 50% does not significantly affect the conclusions. On the other hand, CGL MaxLev 100% to 250% and CGL unconstrained illustrate different risk configurations where volatility is lower than or close to that of the IBrX 100 (Table 2 shows that the market factor volatility is 27.95% p.a., which is practically the same in absolute returns). However, as CGL MaxLev 100% to 250% do not have a similar leverage benchmark to compare with, they are suppressed in Tables 4 and 5 for simplicity.

Table 4 allows us to compare the strategies according to the risk/return ratio. Given that the Sharpe ratio is widely used in the market, despite its limitations, we propose using it initially to compare the portfolios (later, in Table 5, we present another method for ranking the portfolios).

Table 4 shows that over the period 2003-2022, the Sharpe ratios of CGL MaxLev 0% and 50% surpassed 62%, while the benchmarks only reached 43%. Thus, the

CGL model outperformed the risk/return benchmarks in the complete exercise. In addition, CGL MaxLev 0%, which consistently presenting the lowest volatility, offered a higher Sharpe ratio than both or at least one of the unleveraged benchmarks at every four-year interval. Meanwhile, CGL MaxLev 0% and 50% are the only strategies without a negative Sharpe ratio.

These results add to the evidence that the CGL model is a competitive strategy for diversifying factor-based portfolios over shorter investment horizons. Considering, for example, that we are targeting diversified investors, when CGL MaxLev 0% outperforms the Sharpe ratio of the 1/*n* portfolio, it shows that the Fama and French (1993) three-factor model, with the addition of the Carhart (1997) momentum factor, is not sufficient to explain the variations in Brazilian stock returns using only a naïve equal-weights strategy. In other words, the abovementioned results provide additional evidence that it is necessary to apply an active strategy to diversify between such risk factors in Brazil. In addition, when the exercise reveals that the Sharpe ratio from CGL MaxLev 50% outperforms its benchmarks, SR and the Tangency portfolio, it indicates that the CGL model offers a superior strategy to those of its comparable pairs.

Table 4
Sharpe ratio (%)

Panel A	2003-2022			2003-2006			2007-2010		
	Return	Vol.	Sharpe	Return	Vol.	Sharpe	Return	Vol.	Sharpe
Unleveraged single-state models									
1/*n*	14.58	8.06	42.90	26.18	7.49	105.03	15.30	7.63	57.70
SR CONS	14.80	9.99	36.82	24.80	10.93	59.31	11.13	9.29	2.45
Leveraged single-state models									
Tangency	15.05	13.03	30.14	25.12	14.27	47.69	11.26	9.54	3.76
SR	14.54	11.17	30.58	23.53	12.71	41.06	10.54	11.32	-3.14
Unleveraged regime-switching model									
CGL MaxLev 0%	15.65	6.33	71.58	26.86	7.18	119.00	15.83	6.40	77.07
Leveraged regime-switching model									
CGL MaxLev 50%	17.76	10.55	62.92	28.44	11.65	86.91	17.85	10.73	64.81

Panel B	2011-2014			2015-2018			2019-2022		
	Return	Vol.	Sharpe	Return	Vol.	Sharpe	Return	Vol.	Sharpe
Unleveraged single-state models									
1/*n*	5.53	6.90	-60.00	15.27	8.14	54.36	11.62	9.75	55.42
SR CONS	12.88	7.37	43.57	9.37	9.96	-14.88	16.48	11.83	86.74
Leveraged single-state models									
Tangency	14.40	8.47	55.85	7.38	17.87	-19.38	17.87	12.82	90.94
SR UNC	14.31	8.27	56.10	8.47	10.81	-21.95	16.43	12.22	83.54
Unleveraged regime-switching model									
CGL MaxLev 0%	11.20	5.94	25.86	14.22	5.97	56.51	10.85	5.91	78.24
Leveraged regime-switching model									
CGL MaxLev 50%	14.95	10.21	51.74	15.70	10.29	47.17	12.50	9.80	64.07

Notes: *The table shows the Sharpe ratio of the out-of-sample exercise (2003-2022) and in four-year intervals. It shows annualized weekly average returns and volatilities (vol.). Returns are absolute, i.e., not excess returns, and net of transaction costs. CGL MaxLev 0% and 50% use the leverage constraining method introduced by Lewin and Campani (2022). We suppressed the higher leverage CGL model as there is no appropriate leverage comparison. The SR and CGL strategies were optimized for $\gamma = 5$.*
Source: *Prepared by the authors.*

Table 5 indicates the differences in annualized certainty equivalent returns (*ΔCER*) and their confidence intervals. For positive *ΔCERs* and positive confidence intervals, the first line shows that CGL MaxLev 0% outperforms 1/*n* and SR CONS with statistical significance. Similarly, the second line provides evidence for the comparison with leveraged portfolios: CGL MaxLev 50% statistically outperforms the Tangency and SR portfolios.

Table 5
Certainty equivalent returns differences (ΔCER, %)

Regime-switching model	Single-state models			
	1/*n*	**SR CONS**	**Tangency**	**SR**
CGL MaxLev 0%	1.30	2.09	1.90	5.30
	[0.90 - 2.19]	[0.43 - 3.84]	[-1.01 - 2.93]	[-4.54 - 8.86]
CGL MaxLev 50%	3.82	4.22	2.22	5.82
	[0.86 - 6.14]	[-1.75 - 6.71]	[0.27 - 3.49]	[1.14 - 7.78]

Notes: *The table shows the differences between annualized certainty equivalent returns (ΔCER) computed using absolute returns net of transaction costs. The differences correspond to the CER of the models shown in the horizontal panels, minus the CER of the benchmarks in the columns, calculated as in Section 2.7. The horizontal panels show CGL MaxLev 0% and 50%. We suppressed the higher leverage CGL model as there is no appropriate leverage comparison. The SR and CGL strategies were optimized for γ = 5. Below the ΔCER, we report the 95% bootstrap confidence intervals drawn from 1,000,000 samples with replacement using the bias-corrected and accelerated percentile method. The out-of-sample exercise was conducted with weekly observations from Jan/08/2003 to Dec/19/2022.*
Source: *Prepared by the authors.*

4. CONCLUSION

According to Perez-Quiros and Timmermann (2000), Black and McMillan (2004), Guidolin and Timmermann (2008b), Tu (2010), Gulen et al. (2011), and Chincoli and Guidolin (2017), the use of regime-switching models to diversify factor-based portfolios shows promising results. We reinforce this conclusion in an extended setting. In developed markets, the literature presents it in terms of power utility. In emerging markets, it presents it under an even simpler framework, with quadratic utility preferences. Therefore, we propose to bridge the gap in the literature on factor-based investing by using a more realistic utility function: the stochastic recursive utility function. We also focus on a single emerging market, Brazil, due to its regional relevance and to avoid the impact of exchange rates on the estimations.

We constructed an out-of-sample exercise to compare the CGL model with passive and active diversification strategies as benchmarks. As the optimal solution of regime-switching strategies often indicates high leverage, we used Lewin and Campani's (2022) MaxLev procedure to constrain the weights of the CGL portfolio in order to keep the strategy feasible for practical purposes. The procedure was even more necessary to avoid overleveraging, given that factors are intrinsically leveraged, as they long and short their underlying stock portfolios. As a result, we used CGL MaxLev 0% to compare with the unleveraged benchmarks and CGL MaxLev 50% to compare with the leveraged benchmarks. Section 4.3 shows that the 5th percentile, median, and maximum leverage of CGL MaxLev 50% are the closest to the leveraged benchmarks. Thus, it was the most suitable CGL portfolio to compare to them, while setting the maximum leverage between 0% and 50% did not significantly affect our conclusions.

The Sharpe ratios indicate that both leveraged and unleveraged CGL strategies outperform the benchmarks in the full sample. In the shorter subsamples, first, we observe that CGL MaxLev 0% consistently presents the lowest volatility while offering a Sharpe ratio higher than at least one of the unleveraged benchmarks. Second, unlike all the benchmarks, the Sharpe ratios of the leveraged and unleveraged CGL strategies are positive in every subsample examined. Finally, as a robustness check, the certainty equivalent returns reveal that both CGL strategies outperform their benchmarks with statistical significance.

The current research shows that regime-switching models provide competitive strategies for efficiently diversifying factor-based portfolios in the Brazilian stock market. Further studies can include more factors and other markets. Another path for further investigation is to use a higher data frequency to update the factors more frequently. For example, quick updates to sort small, neutral, and big stocks might produce an even better momentum strategy. We highlight that sophisticated investors and fund managers could apply the methodology used in this article to create advanced risk factor-based strategies that can benefit from active management. In doing so, they will be likely to outperform all, or at least most, of the factors over the long run.

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