Revista de Contabilidade e Organizações
www.rco.usp.br

DOI: http://dx.doi.org/10.11606/rco.v11i29.125503

Journal of Accounting and Organizations
www.rco.usp.br

Comparative analysis of different risk metrics in the composition of a fund of real estate investment funds

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Article information

Article history
Received: January 12, 2017
Accepted: May 20, 2017

Keywords:
Real estate investment funds.
Conditional value at risk.
Investment analysis.
Portfolio optimization.

Abstract

The main purpose of this study is to analyze comparatively the results of different risk metrics in the composition of a Fund of Real Estate Investment Funds (FIIs in the Portuguese acronym). From the revision of literature, one chose to adopt the risk metrics variance and Conditional Value at Risk (CVaR), developed in the studies of Markowitz (1952) and Rockafellar and Uryasev (2000, 2002), respectively. The study encompassed a sample of 30 FIIs listed on BMF&BOVESPA stock exchange, with daily return between July 2013 and July 2015. As methodology, modeling and computational simulation of theoretical portfolios for minimization of non-systematic risk were used, submitting them to restrictions according to risk profiles and regulation instructions. The main confirmation is that different metrics produced different results in the portfolios of lower risk level. In this situation, the CVaR minimized more accentuated losses, coming from its focus on downside risk. The risk metric variance, in its turn, underestimated the probability of events arising from tail risk, being sub-optimal for investors whose primary objective is minimization of risk, even if with lower expected return potential. However, from determined risk level, both metrics equal their results. These results could be verified in all efficient frontiers of the funds analyzed.

1. INTRODUCTION

Individuals, companies and financial institutions face problems in the resource allocation among numerous alternatives of investments on the capital market. As consequence, the following questions naturally emerge: how to maximize the assets with exposition to the lowest risk possible? The studies of Markowitz (1952, 1959) are the genesis of what today is called Modern Portfolio Theory (MPT) or Mean-Variance Analysis, when, for the first time, one studied the risk and return behavior of sets of assets rather than separate assets, and there was the emergence of concepts such as diversification and efficient portfolios.

Recent studies developed regarding MPT are related to the incorporation of transactions costs, greater precision in estimating parameters, new methods of diversification and multiperiod optimizations (Kolm, Tütüncü, & Fabozzi, 2014). However, some MPT premises started being questioned in the 1990's: is variance the most appropriate risk measure? Are all returns of assets normally distributed? From these questionings, a new concept of risk appears: downside risk. In this methodology, the target of study starts being the tails of losses of returns distribution. These new studies are also called Post-Modern Portfolio Theory (PMPT), considered an extension of MPT (Rom & Ferguson, 1994).

Both methodologies may be allocated in the analysis of a new class of assets on the Brazilian capital market, the Real Estate Investment Funds (FIIs), which have attracted attention mainly from individual investors. Although created in 1993, the sector grew significantly only after a series of new regulations and incentives, at the end of the 2000's. Following the high growth of civil construction in the period between 2008 and 2013, the issuance of FIIs grew by more than 250% in the same period, from 73 to 257 (Brazilian Financial and Capital Markets Association [ANBIMA], 2014).

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Due to the complexity that may be generated by the analysis of this significant number of funds for investors, the strategic adoption of the Funds of Funds consolidates a viable and balanced solution in confrontation of risks and returns.

They are also called multi manager funds, that is, they are investment funds that invest directly in quotas of other investment funds. This strategy allows investors to access a highly diversified portfolio by acquiring only one asset, not requiring individual analysis for decision-making regarding the diversification and composition of their own fund portfolio (Strachman & Bookbinder, 2010). Within this context, the objective of this research is to analyze, in a comparative way, variance risk and downside risk in optimized portfolios of Funds of FIIs.

2. THEORETICAL REVISION

2.1 Risk diversification

The assets can be classified into two simple segmentations: with risk or risk-free. A risk-free asset is defined as that on which the investor is sure about the return expected, that is, its risk variance is zero (Reily & Brown, 2011; Vernimmen, Quiry, Dallocchio, Le Fur, & Salvi, 2014).

On the other hand, in the risk analysis present in risk assets, several risk sources are noticed: operational, financial, liquidity, exchange rate, market, country. But, an important difference can be outlined: the risks called unsystematic (risks specified in the asset to be analyzed) are susceptible to diversification, while systematic risk (market risk) cannot be eliminated by diversification. The primary analyses of this phenomenon were developed by Markowitz (1952, 1959) (Bodie, Kane, & Marcus, 2011; Reilly & Brown, 2011; Berk, Demarzo, & Harford, 2012). Figure 1 presents situations of variance of systematic (non-diversifiable) and unsystematic (diversifiable) risk of the portfolio depending on the quantity of assets allocated.

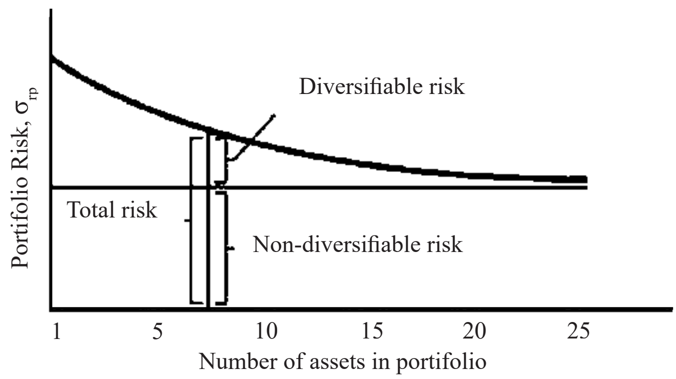

Figure 1. Effect of diversification on the portfolio risk
Source: Gitman (2010)

Thus, portfolios with allocation concentrated in a few assets have high total risk, given that the impact of the unsystematic portion of risk of these respective assets on the portfolio was not effectively minimized through diversification. From the incorporation of more assets, the unsystematic risk is susceptible of being gradually minimized, with the global risk of the portfolio reaching levels close to the systematic risk, this being common to all risk assets.

Harry Markowitz, at the time of publication of his revolutionary article "Portfolio Selection" on the Journal of Finance, in 1952, learned that variance of an asset, alone, was not relevant. He noticed that the variance of this asset in a portfolio is that which brings greater success for the investment analysis. Variance and return expected on a portfolio without short sale are presented in Equations (1) and (2) (Markowitz, 1952; Elton, Gruber, Brown & Goetzmann, 2012).

$$\sigma_P^2 = \sum_{j=1}^{N} X_j^2 \sigma_j^2 + \sum_{j=1}^{N} \sum_{\substack{k=1 \\ k \neq j}}^{N} \sigma_{j,k} X_j X_k \quad (1)$$

$$\overline{R_P} = \sum^{N} X_i \bar{R}_i \quad (2)$$

Where: $\overline{R_P}$= Return expected on the portfolio; σ_P^2= Portfolio variance; $\overline{R_i}$= Return expected on asset i; X_i= Weight of the asset *i*; X_j= Weight of the asset *j*; X_k=Weight of the asset *k*; σ_j^2= Variance of asset *j*; $\sigma_{j,k}$= Co-variance of assets *j* and *k*; N= Number of assets.

Studies on diversification could already be found in the beginning of the 20th Century, but the novelty brought by Markowitz (1952, 1959) approached how the risk of a portfolio was connected with the co-variances of the individual assets that compound it. Thus, the author consolidated what was conventional knowledge in investments, also generating a process by which investors could choose optimized diversified portfolios: the Mean-Variance Analysis (Damodaran, 2007).

2.2 Mean-Variance Analysis

The methodology of selection of portfolios proposed by Markowitz (1952, 1959) is concerned about two dimensions: return expected on the portfolio and variance of the returns of the portfolio. All possible combinations of assets available to the investor compound the feasible set of portfolios. Nevertheless, only portfolios that offer the greatest expected return to determined risk are efficient portfolios. Consequently, there is an efficient portfolio for each risk level. The set of these efficient portfolios is called efficient frontier (Fabozzi & Markowitz, 2011), equivalent to the example in Figure 2 below:

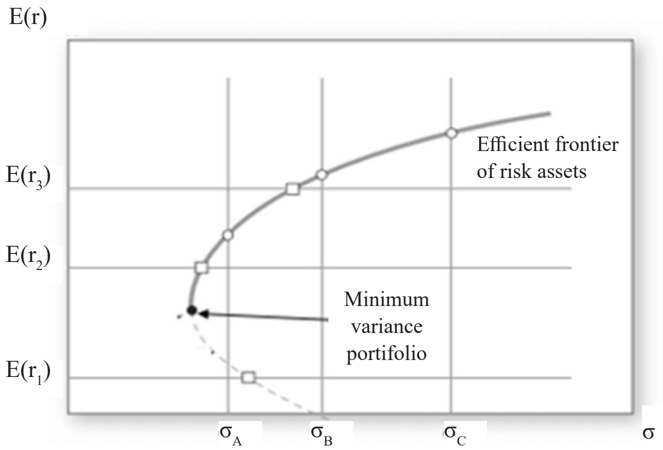

Figure 2. Efficient frontier without free-risk asset
Source: Adapted from Bodie, Kane and Marcus (2011)
Note: σ_A =Portfolio A Risk; σ_B = Portfolio B Risk; σ_C = Portfolio C Risk;
$E(r_1)$ = Expected Return on the Portfolio 1; $E(r_2)$ = Expected Return on Portfolio 2;
$E(r_3)$ = Expected Return on Portfolio 3

If a risk-free asset is introduced as investment option, the efficient frontier will become a line that reaches the efficient frontier of risk assets. The line is called Capital Market Line and represents the options of compositions of the risk-free asset with a portfolio that represents the capital market as a whole (Fabozzi & Markowitz, 2011). The efficient frontier with the risk-free asset is presented in Figure 3.

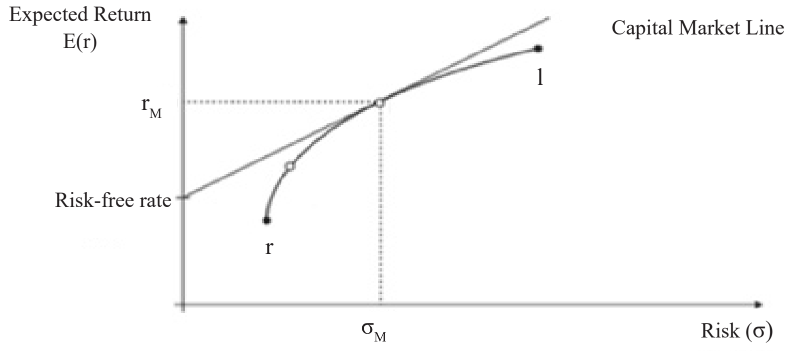

Figure 3. Efficient frontier with free-risk asset
Source: Adapted from Vernimmen et al. (2014).
Note: σ_M = Risk of Market Portfolio; r_M = Expected Return of Market Portfolio

Mean-Variance methodology has two basic premises: the normality of distribution of return of the assets; and, the functions of utility of all investors are quadratic. Damoradan (2007) refutes the premise on returns, affirming that most of the investments do not have normally distributed and symmetric returns. Elton and Gruber (1997) corroborate the configuration of this method by showing how Mean-Variance Analysis is sensitive, presenting dependence on the way the analysis inputs (expected return and variance of the assets) are calculated. They also affirm that the size of historical series to be used is a relevant matter and little research has been performed on the subject.

2. 3 VALUE AT RISK AND CONDITIONAL VALUE AT RISK

Value at Risk (VaR) and its adaptation Conditional Value-at-Risk (CVaR) are measures of downside risk. These approaches are concerned about the magnitude of possible losses by the investor (Dempster, 2002). The terminology downside risk was firstly studied by Roy (1952) in the sequence of publications of Harry Markowitz on Mean-Variance. Roy (1952) criticized the function utilities and the obscurity of foreseeing the investors' utility, proposing that investors prefer investments with the smallest chances of extreme losses (Nawrocki, 1999, as mentioned in Araújo, 2011). Jorion (2006) defines VaR as a measure of the greatest potential loss in value of a risk asset or portfolio at a given period of time for a given confidence interval. The non-parametric mathematical formulation of VaR is shown in Equation (3).

$$1 - c = \int_{-\infty}^{VaR} f(w)dw = P(w \leq VaR) \tag{3}$$

Where: *c* = Confidence Interval; f(w) = Density of probability of the future value of the portfolio.

The possibilities of f(w) specification are numerous, and each one has its premises, advantages and disadvantages (Damodaran, 2007). Artzner, Delbaen, Eber and Heath (1997, 1999), additionally, propose a set of axioms that constitute properties for a risk measure to be considered coherent. The axioms are: monotonicity, subadditivity, positive homogeneity and translation invariance (Acerbi & Tasche, 2002). Thus, in spite of the great success of VaR, criticism appeared in the late 1990's, there being two main points indicated: VaR measures only the percentiles of distribution of losses, leaving losses beyond the VaR level aside (tail risk); and, it is not a coherent risk measure, since it is not subadditive (Artzner et al., 1997, 1999; Yamai & Yoshiba, 2005).

Acerbi (2002) points that subadditivity is essential for a risk measure, since this axiom captures the essence of how a portfolio made from sub-portfolios will have its risk measured. With the violation of subadditivity, the risk of a portfolio might be greater than the sum of its parts, bringing non-optimal results for the risk management.

To overcome the problem of tail risk and subadditivity, Artzner et al. (1997, 1999) propose a new risk measure: the CVaR (also called expected shortfall, tail Value at Risk and mean shortfall). Satisfying all axioms of a coherent risk measure, the CVaR can be defined as the weighted median of the losses expectations when these are greater than the VaR (Moreira, 2006). The formulation of the CVaR in the Equation (4) and the comparison of the properties of VaR and CVaR is presented in Table 1.

$$CVaR^{\alpha}=E(P|P>VaR^{\alpha}) \quad (4)$$

Where: α = Confidence Interval; P = Profit/loss of the portfolio; VaR^{α} = VaR with confidence interval.

Table 1. Comparison between VaR and CVaR

Property	VaR	CVar
Monotonicity	Yes	Yes
Subadditivity	No	Yes
Positive homogeneity	Yes	Yes
Translation Invariance	Yes	Yes

Source: Elaborated by the authors

Posteriorly, Rockafellar and Uryasev (2000, 2002) made advances in optimization involving the CVaR, reaching the conclusion that the problem may be solved by stochastic linear programming. The approach consists in simultaneous calculations of CVaR and VaR, observing that the optimization of the CVaR also implies in optimization of the VaR (because CVaR is always greater than or equal to VaR). Recent studies developed on CVaR indicate that the new metric is more sensitive than VaR regarding errors in the estimation of inputs. Thus, the success of CVaR is highly connected to the precision of modeling of distribution of risk above the VaR (tail risk) (Uryasev, 2010).

2.4 Real Estate Investment Funds

The FIIs were created by Law 8,688/93, with the Instructions of the Securities and Exchange Commission (CVM) 205/94 and 206/94, bringing the guidelines and rules for functioning and distribution. The reference at the time was the REITs in the United States, even though the FIIs did not absorb all its characteristics (Decree 8,688 of June 25th, 1993, & ANBIMA, 2014). The last revision of the regulation defines the FIIs as: "*a communion of funds raised through the system of distribution of securities and intended for application in real estate development. The fund shall be constituted as closed-end condominium and shall have undetermined validity term.*" (CVM Instruction no. 472, 2008, p. 1).

The profits coming from FIIs distributed to individual investors are also exempt from income tax, provided that 95% profits are distributed to the shareholders; there is a minimal of 50 shareholders, and no individual shareholder has 10% shares of the fund; and no related party (construction company, real estate developers) has more than 25% shares of the fund (CVM Instruction no. 472, 2008).

Regarding their configuration, the FIIs are classified as closed-end investments funds. Thus, for an investor to sell his/her share (ideal FII fraction), access to secondary market is necessary. As a rule, all FIIs must have negotiation either on the stock exchange or on organized over-the-counter market. Such funds can also be classified according to the assets that compound their portfolio: Income Funds, Development Funds and Financial Assets Funds.

The Real Estate Index Fund (IFIX in the Portuguese acronym), in its turn, is a market sectorial index created by BM&BOVESPA, aimed at being the indicator of average performance of the FIIs listed on the stock market and on organized over-the-counter market. Thus, the IFIX reflects the dynamics of the market as a whole, being the first benchmark of the sector (BM&FBOVESPA, 2015; UQBAR, 2015). It was created in September 2012 and its historical series can be followed retroactively since December 2010, when the index was fixed at 1,000 points by its creator.

Being a total return index, the IFIX does not only reflect the variation in shares price but also dividends, interest on shareholders' equity, subscription rights and bonuses. Thus, the total value of the IFIX theoretical portfolio is composed of "ex-theoretical" prices of the assets that compound it (BM&FBOVESPA, 2015).

2.5 Real Estate Investment Trusts (REIT) Indicators

North American REITs were reference for the FIIs creation. Their conception goes back to the end of the 19th Century, but it was in 1960 that the US Congress approved the legislation necessary to its regulation, seeking to stimulate and pulverize new investments in real estate industry (Brueggeman & Fisher, 2008; Calado, Giotto, & Securato, 2001).

They can be classified into three major types: Equity REITs, Mortgage REITs (investment through debt) and Hybrid REITs (mix between Equity and Mortgage types). At the end of 2014, there were 203 REITs with combined market capitalization of US$ 830 billion (FTSE, 2015; Block, 2006). Despite the similarities, FIIs and REITs have some differences: REITs are incorporated as companies, they must have at least 100 shareholders, 5 major shareholders cannot have more than 50% REIT share and 90% profits must be distributed (Cosentino & Alencar, 2011).

Block (2006) and Brueggeman and Fisher (2008) mention two important indicators to evaluate the REITs: Price/Funds from Operations (P/FFO) and Net Asset Value Premium (P/NAV). FFO, a non-accounting measure, is used as a cash flow metric generated by the REIT available for distribution to shareholders. FFO is preferable (rather than Net Income) due to the impact of depreciation and non-recurring revenues on cash flow. The multiple market value of the REIT divided by FFO (or share price divided by FFO per share) provides a comparative analysis with similar REITs. The formula for FFO and the multiple P/FFO are found in Equations (5) and (6), respectively:

$$\text{FFO} = \text{Net Income} + \text{Depreciation} - \text{Profit (Loss) in Assets Sale} \quad (5)$$

$$\text{P/FFO} = \frac{\text{REIT Market Value}}{\text{FFO}} \quad (6)$$

$$\text{NAV} = \text{REIT properties market value - Debt} \quad (7)$$

Net Asset Value (NAV) is the market value estimated of all REIT assets after all obligations and liabilities are subtracted. It should be estimated by analysts and investors since the REITs do not have periodic evaluations of prices of market of the properties that compound their portfolios (Block, 2006). The P/NAV seeks to evaluate if the REITs assets are over or sub evaluated. The formula for NAV and NAV-P are presented in Equations (7) and (8), respectively:

$$\text{P/NAV} = \frac{\text{REIT Market Value}}{\text{Net Asset Value}} \quad (8)$$

3. METHODOLOGY

3.1 Determination of the models

To conduct the comparison between the two different types of models, risk variance and downside risk, one chose the model of Markowitz (1952) for risk variance, and the optimization of the CVaR proposed by Rockafellar and Uryasev (2000, 2002) for downside risk. The CVaR was preferable to VaR for having mathematic properties more favorable to the optimization and importance of tail risk, according to what is presented in Table 1. The objective functions and restrictions of the model are presented in Table 2.

Table 2. Models used for comparison

Markowitz (1952)	Rockafellar and Uryasev (2000, 2002)
To minimize:	To minimize:
$\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$	$\alpha + \frac{1}{q(1-\beta)}\sum_{k=1}^{q}[-w^T y_k - \alpha]^+$
Subject to:	Subject to:
$\sum_{i=1}^{n} w_i = 1$	$\sum_{i=1}^{n} w_i = 1$
$\sum_{i=1}^{n} w_i R_i = R_w$	$\sum_{i=1}^{n} w_i R_i = R_w$
$0 \leq w_i \leq 1$	$0 \leq w_i \leq 1$

Source: Markowitz (1952) and Rockafellar and Uryasev (2000, 2002)

where: wi = Weighting of the asset “I” in the portfolio; σij = Co-variance of assets “I” and “j”; Ri = Expected return on asset “I”; Riw = expected return of the portfolio; β = probability level of maximum losses; α = Maximum Value at Risk; yk = Vector of losses of scenario “k”; q = Number of scenarios.

3.2 Sampling and period analyzed

After defining the models to be implemented, the next step is to define the assets that will be eligible for the composition of the Fund of FIIs, and the study period. As guiding principles of eligibility, one has looked for liquidity offered by the FII, the nature of its investments and availability of data. On July 18th, 2015, the IFIX, index that measures the performance of the FIIs plus liquid funds negotiated on the scope of BM&FBOVESPA, had in its theoretical portfolio 46 FIIs that are negotiated on the stock market. Among these, three FIIs are funds of FIIs that will not be considered in the sample.

Analyzing data availability (condition of at least 90% quotations present in the period), previous studies and literature review, it was decided to use the study period of two years, from July 2013 to July 2015. Adjusted closing prices for dividends and income were accessed through Economatica software database. Of the forty-three FIIs selected, only thirty have more than 90% daily quotations in the period, which compound 66.2% IFIX. Finally, the eligible FIIs and the distribution per class of asset are presented in Table 3.

Table 3. Eligible FIIs

Code	Name	Class
AEFI11	AESAPAR	Schools
AGCX11	Agências Caixa	Bank Branches
BBPO11	BB Progressivo II	Bank Branches
BBRC11	BB Renda Corporativa	Bank Branches
BBVJ11	BB Votorantim JHSF Cidade Jardim Continental Tower	Offices
BRCR11	BTG Pactual Corporate Office	Offices
FFCI11	Rio Bravo Renda Corporativa	Offices
FIGS11	General Shopping Ativo e Renda	Shopping Malls
FLMA11	Continental Square Faria Lima	Offices
HGBS11	CSHG Brasil Shopping	Shopping Malls
HGLG11	CSHG Logística	Logistics
HGRE11	CSHG Real Estate	Offices
KNCR11	Kinea Rendimentos Imobiliários	Financial Assets
KNRI11	Kinea Renda Imobiliária	Logistics
MXRF11	Maxi Renda	Financial Assets
PQDP11	Parque Dom Pedro Shopping Malls	Shopping Malls
PRSV11	Presidente Vargas	Offices
RBGS11	RB Capital General Shopping Sulacap	Shopping Malls
RBRD11	RB Capital Renda II	Logistics
RNGO11	Rio Negro	Offices
SAAG11	Santander Branches	Bank Branches
SDIL11	SDI Logística RIO	Logistics
SPTW11	SP Downtown	Offices
TBOF11	TB Office	Offices
TRXL11	TRX Realty Logística Renda I	Logistics
VLOL11	Vila Olímpia Corporate	Offices
VRTA11	Fator Verita	Financial Assets
XPCM11	XP Corporate Macaé	Offices
XPGA11	XP Gaia Lote I	Financial Assets
XTED11	TRX Edifícios Corporativos	Offices

Source: Elaborated by the authors

3.3 Data treatment

After defining the eligible FIIs, return interval and study period, the next step is to export the adjusted daily closing quotes from Economatica software to a spreadsheet using Microsoft Excel software. The quotations exported were from July 19, 2013 to July 20, 2015, consisting of 496 trading days. From the quotations, the daily returns were obtained by Equation (2). In some days, some assets were not traded and it was not possible to calculate their returns. To overcome this obstacle, on the dates when there were no quotations, the prices of the immediately preceding day were maintained.

For the application of the REITs indicators to the FIIs, the last audited financial statements of each FII registered on BMF&BOVESPA system were used. Due to legal peculiarities and differences in corporate structures, some adaptations in the calculation methods were performed. As the FIIs are not allowed to depreciate their assets, all non-recurring income and expenses, i.e., those related to sales of assets (except when it is the investment objective of the FII) and adjustment in value of real estate were discounted.

The NAV of each FII was estimated by its owners' equity.

The calculation methods adopted for the indicators applied to the FIIs are presented in Equations (9) and (10).

$$\text{FFO} = \text{Net Profit-Profit (Loss) with Sale of Assets - Profit (Loss)} \quad \text{in adjustment of the Real Estate Value} \tag{9}$$

$$\text{NAV} = \text{Assets - Liabilities} \tag{10}$$

3.4 Restrictions

Restrictions regarding the composition of the portfolios define the limits of the relative weights of each asset allowed for optimization. In the study, four types of restrictions were performed, all without the permission of short sales.

3.4.1. All assets

Two portfolios are present in this classification: “Global” Fund and “Limits per Issuer” Fund. The first allows the assets to have any weight between zero and one, while the latter follows the limits imposed by the CVM regarding the concentration of the net equity of the fund when the issuer is another investment fund (CVM Instruction no. 409, 2004).

3.4.2 Restrictions related to beta

In this category, Beta in relation to IFIX acts as a divisor between FIIs with different levels of risk. The separation was performed by means of the median of Beta coefficients of all eligible FIIs. Two funds compound this type of restriction: “Beta < Median” Fund and “Beta > Median” Fund.

3.4.3 Restrictions related to FIIs indicators

With the P/FFO and P/NAV indicators, the FIIs were divided into groups according to the median of each indicator. The market capitalizations used for the calculation of the P/FFO and P/NAV were the last contained in the historical series used for the optimization, the capitalizations of July 20, 2015. Table 4 presents the indicator values of each FII.

Table 4. P/FFO and P/NAV

FII	P/FFO	P/NAV	FII	P/FFO	P/NAV
BRCR11	9.8	0.7	VLOL11	9.3	0.6
AEFI11	8.6	0.7	PRSV11	5.2	0.5
AGCX11	10.9	0.9	RBGS11	6.0	0.5
BBRC11	34.1	0.9	RBRD11	9.3	1.0
BBPO11	10.2	1.1	FFCI11	10.2	0.8
BBVJ11	11.4	0.5	RNGO11	8.3	0.9
HGLG11	16.1	1.0	FLMA11	11.0	0.6
HGBS11	10.8	0.8	SAAG11	10.3	0.9
PQDP11	6.1	0.6	SDIL11	8.6	0.8
VRTA11	11.0	1.0	SPTW11	66.0	0.7
FIGS11	7.6	0.7	TBOF11	8.6	0.6
HGRE11	13.6	0.9	TRXL11	7.7	0.7
KNRI11	11.1	0.8	XTED11	5.8	0.6
KNCR11	17.5	1.6	XPGA11	3.6	0.9
MXRF11	10.0	0.9	XPCM11	11.9	0.8
Median	**10.1**	**0.8**			

Source: Elaborated by the authors

3.4.4 Restrictions related to the class of asset

The restriction per class of asset is composed of an isonomic fund between the different types of existing real estate development projects. For each class, a representative FII was defined, and these assets were selected according to their risk and return relationship. The portfolio was called “Iso” Fund.

3.5 Optimization of models

With the data properly organized and the restrictions predetermined, the next step was to optimize the models. The expected return on each FII was calculated by the historical median of the returns. Both Mean-CVaR (95% confidence interval) and Mean-Variance optimizations were performed by MathWorks MATLAB software. For each of the optimizations, two different types of solver are used: stochastic programming for Mean-CVaR and quadratic programming for Mean-Variance (MathWorks, 2015). To compare the different risk metrics, the portfolios optimized by Mean-CVaR had their standard deviations calculated. From the data obtained for each portfolio of each restriction (weights, expected returns and standard deviations), efficient frontiers were plotted using Microsoft Excel software.

4. RESULTS AND DISCUSSION

In this chapter, one presents the efficient frontiers for the various restrictions calculated from the Mean-CVaR and Mean-Variance optimizations with historical quotations of the 30 eligible FIIs from July 2013 to July 2015.

4.1. All assets

Figure 4 presents the efficient frontiers for the "Global" Fund, unrestricted. Some FIIs selected for reference are also plotted. The selection sought to identify the FIIs with the best risk and return relationships, ratio formulated in Equation (5). The Minimum Variance Portfolio (MVP) is also highlighted.

$$RR_i = \frac{R_i}{\sigma_i} \quad (5)$$

where: RR_i = Risk-Return Ratio; $\overline{R_i}$= Expected return on asset "i"; σ_i = Standard deviation of asset "i"

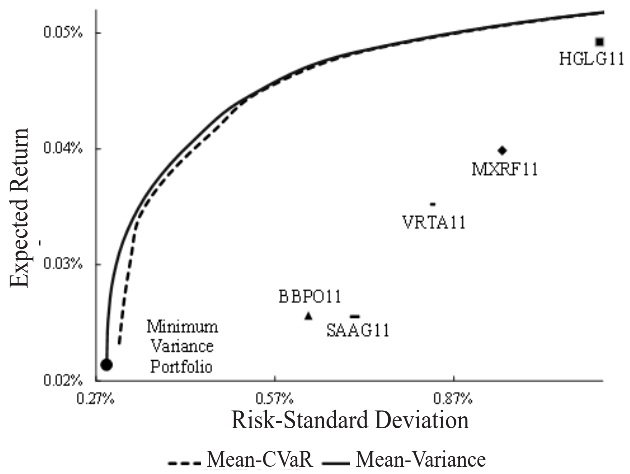

Figure 4. Efficient Frontiers of the "Global" Fund
Source: Elaborated by the authors

All optimized portfolios are efficient, that is, they have a higher return for the same risk than the IFIX theoretical portfolio. The outstanding difference between Mean-CVaR optimization and Mean-Variance is the frontier discrepancy at low risk levels. This occurs because the Mean-CVaR optimization seeks to reshape extreme losses, while Mean-Variance treats them as a normal distribution.

When comparing portfolios of the same expected return, the Mean-Variance optimization provides portfolios with up to 10% less variance than Mean-CVaR portfolios. This difference tends to disappear according as the risk level increases, suggesting that the Mean-Variance optimization underestimates the chances of large losses (tail risk). Similarly, the efficient frontiers of the "Limits per Issuer" Fund are shown in Figure 5.

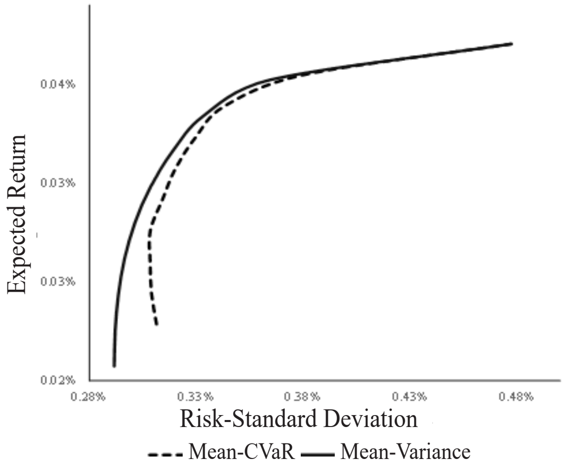

Figure 5. Efficient Frontiers of the "Limits per Issuer" Fund
Source: Elaborated by the authors

Like the "Global" Fund, the apparent differences between risk metrics are most notable at lower risk levels. With the imposition of regulatory limits by issuer, the portfolios have a lower capacity to receive high returns. The maximum annual return with the "Limits per Issuer" Fund is 10.89% compared to the possible 16.35% with the "Global" Fund. It is also possible to observe that some portfolios obtained by Mean-CVaR are not efficient from the risk variance point of view.

4.2 Restrictions related to beta

Figure 6 presents four efficient frontiers: Mean-CVaR and Mean-Variance of "Beta <Median" and "Beta> Median" Funds.

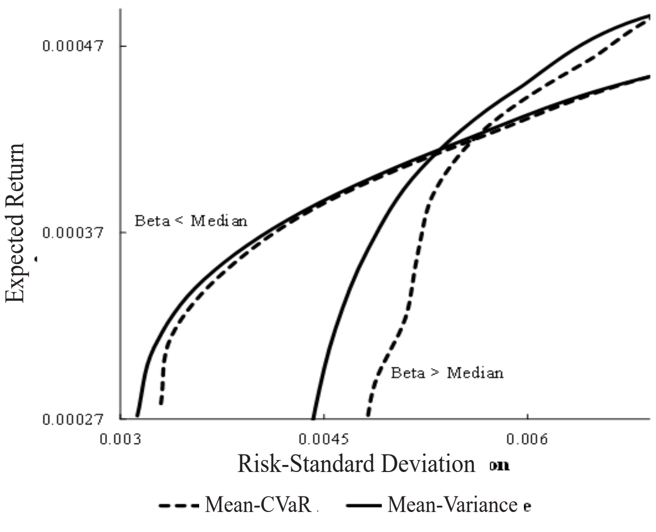

Figure 6. Efficient frontiers of the "Beta <Median" and "Beta> Median" Funds
Source: Elaborated by the authors

The points of intersection represent when one frontier becomes dominant over the other. Thus, an investor who initially sought to invest in the "Beta <Median" Fund frontier, would prefer to switch to the "Beta> Median" Fund frontier from the level of risk at which the former will intersect with the latter (related to the two metrics under analysis). The optimizations present greater differences between them when the assets have larger Betas, reaching 15% of relative difference for the same expected return.

4. 3 RESTRICTIONS RELATED TO FIIS INDICATORS

The efficient frontiers for the “Growth” and “Discount” Funds were plotted in the same chart for comparison. Each one of the restrictions has 15 FIIs. The frontiers are presented in Figure 7.

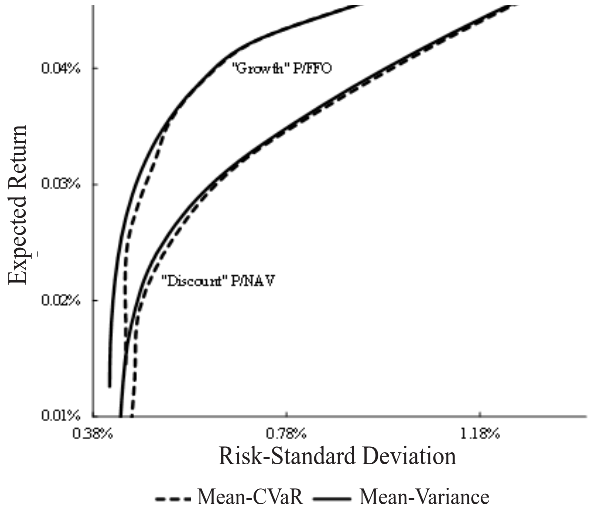

Figure 7. Efficient Frontiers of the “Growth” and “Discount” Funds
Source: Elaborated by the authors

Portfolios comprised of assets of the “Growth” Fund were always efficient in comparison with those of the “Discount” Fund, even if higher expected returns could be achieved by the “Discount” Fund. Thus, it is evident that the P/FFO indicator appears to be more suitable for the evaluation of FIIs, rather than the P/NAV indicator. However, the relative difference between the two metrics was slightly accentuated for the “Growth” Fund portfolios, reaching 8% for the same expected return at the lowest risk level.

4.4 Restrictions related to the class of asset

The “Iso” Fund was composed of a FII of each class, namely: Logistics, Bank Branches, Financial Assets, Offices, Shopping Malls and Schools. Assets were selected by the best risk and return relationships, according to Equation (5). The efficient frontier, the points representing eligible FIIs, and the IFIX index are shown in Figure 8.

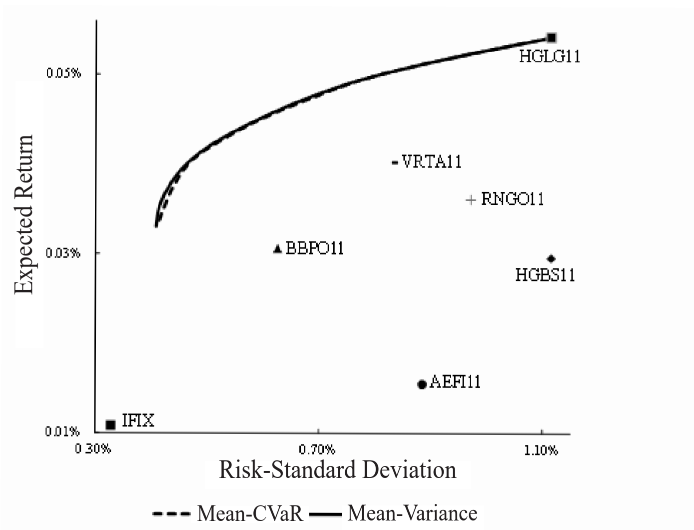

Figure 8. Efficient frontiers of the “Iso” Fund
Source: Elaborated by the authors

Only the IFIs of the "Iso" Fund could not obtain a portfolio with risk level of the IFIX. The MVP of this restriction has daily standard deviation of 0.41%, while the IFIX has 0.33%. However, the portfolios of this efficient frontier were able to produce larger returns for the same risk ratio, compared with the portfolios of the "Limits per Issuer", "Growth" and "Discount" Funds. It should be noted that this restriction was the one that presented the least differences between optimizations, with a maximum relative difference of 3.12% risk for the same expected return.

5. FINAL CONSIDERATIONS

The main objective of this work was the comparative analysis of different risk metrics in the optimization of a Fund of FIIs, and models of MPT and PMPT were used. The methodology used involved the survey of the main models used, such as market risk metrics, sample selection and analysis period, and additional restrictions to the models. We analyzed 30 FIIs listed on BM & FBOVESPA stock exchange between July 2013 and July 2015, comprising 496 trading days.

It was observed that at lower risk levels, CVaR and Variance risk metrics presented different results. Because it is a downside risk metric, CVaR seeks to minimize extreme losses (left tail, tail risk). Thus, even with expected lower return in the comparison with the Mean-Variance optimization (when comparing the portfolios by their respective standard deviations and expected return), it is more suitable for investors interested in the lowest risks possible, since tail risk is minimized - although with lower expected return potential. These losses were underestimated by the variance, since this assumes the normal distribution of returns. Thus, for investors more interested in investing in lower risk levels, the composition of a Fund of FIIs is suggested from the portfolios generated by the Mean-CVaR optimization. From a certain level of risk, both metrics converge to the same results.

For future studies, it is recommended to analyze the impact of other types of real estate securities on the performance of the FII Funds. It is also suggested, when available, the analysis with longer study period and larger FIIs sampling, in order to characterize the behavior of the returns distributions of the FIIs in a deeper way.

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