Portfolio construction and risk management: theory versus practice

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Abstract

Purpose – This paper aims to identify a possible mismatch between the theory found in academic research and the practices of investment managers in Brazil.

Design/methodology/approach – The chosen approach is a field survey. This paper considers 78 survey responses from 274 asset management companies. Data obtained are analyzed using independence tests between two variables and multiple regressions.

Findings – The results show that most Brazilian investment managers have not adopted current best practices recommended by the financial academic literature and that there is a significant gap between academic recommendations and asset management practices. The modern portfolio theory is still more widely used than the post-modern portfolio theory, and quantitative portfolio optimization is less often used than the simple rule of defining a maximum concentration limit for any single asset. Moreover, the results show that the normal distribution is used more than parametrical distributions with asymmetry and kurtosis to estimate value at risk, among other findings.

Originality/value – This study may be considered a pioneering work in portfolio construction, risk management and performance evaluation in Brazil. Although academia in Brazil and abroad has thoroughly researched portfolio construction, risk management and performance evaluation, little is known about the actual implementation and utilization of this research by Brazilian practitioners.

Keywords Risk, Performance evaluation, Portfolio construction

Paper type Research paper

1. Introduction

Many new methods and concepts have emerged in financial portfolio construction, risk management and performance evaluation since Markowitz's (1952) pioneering work. The use of the variance, or standard deviation, of returns as a proxy for investment risk has been questioned, and alternative risk measures have been proposed, according to Rom and Ferguson (1994); Roman and Mitra (2009) and Araújo and Montini (2015). The use of the original sample covariance matrix to estimate the risk of an asset portfolio has also been questioned, according to Chan *et al.* (1999) and

Santos and Tessari (2012). Not even the use of quantitative methods to optimize portfolios or to allocate resources among different asset classes has been spared from criticism (DeMiguel *et al.* (2009).

Given the vast academic literature that proposes, explains and details numerous methods for portfolio construction and risk management and performance evaluation, the following central research question is addressed in this paper: among the major quantitative techniques for portfolio construction, risk management and performance evaluation suggested by the international and Brazilian academic literature, which ones are actually adopted by financial market practitioners? The main objective is to identify possible differences between what is taught in classrooms or discussed in academic conferences and what is involved in the day-to-day practice of asset managers and to determine whether a mismatch exists.

In addition to the central research question, the present work aims to compare Brazilian data with European data to inform us about the degree of globalization of the Brazilian asset management industry. Another specific goal is to check the existence of different practices among subgroups in the industry, such as companies of foreign and Brazilian origin, and to investigate the determinants of optimization and risk budgeting.

To achieve these objectives, we conduct a literature review and obtain 78 responses from a field survey with 274 asset management companies. The number of respondents is considerable, especially given the limited universe of professional asset management companies and the difficulty of extracting information from participants in a highly competitive and results-oriented segment. The field survey used an online questionnaire similar to that applied by Amenc *et al.* (2011) in Europe. To analyze the data, the present paper uses Pearson's chi-square independence test, multiple regressions using ordinary least squares (OLS) and a probit model.

According to Amenc *et al.* (2011), little is known about the dissemination and use of academic research findings among investment professionals. Survey-based papers can assist in guiding and developing empirical studies and new theories by academics and can alert practitioners of research-informed recommendations that have not yet been adopted, according to Graham and Harvey (2001). However, similar surveys for Brazil have not been found, thus confirming Amenc *et al.*'s study (2011) and justifying this paper.

This paper is divided into five sections, including the introduction. In Section 2, based on the existing theory, we formulate hypotheses regarding the practices of investment managers. Section 3 describes the empirical methodology of both the field survey and data analysis. Section 4 presents the results, and Section 5 provides final remarks and suggestions for future studies.

2. Literature review and Hypotheses

2.1 Measuring market risk

This paper uses the definition of market risk proposed by Dowd (2002): it is the risk resulting from changes in prices, such as changes in public companies' share quotes, and rates, such as the interest rate and exchange rate. Two ways to quantify this risk are using variance and using downside risk.

Markowitz (1952), who used the variance of returns as a measure of risk, was one of the pioneers in proposing a quantitative methodology for portfolio construction. His work, along with that of Sharpe (1964) and Lintner (1956), generated discussions and publications that formed the modern theory of portfolios.

Rom and Ferguson (1994) and Roman and Mitra (2009) argued in favor of more appropriate risk measures for asymmetric distributions of returns, so-called downside risk metrics. They proposed a new paradigm for portfolio construction and risk management, made possible by computational advances and made necessary by the increasing use of derivatives in portfolios. They termed this new paradigm the post-modern portfolio theory.

Among the downside risk metrics, semivariance, proposed by Markowitz (1959), is equivalent to the variance using only below-average returns. Bawa (1975) and Fishburn (1977) subsequently developed lower partial moments (LPM). Series can be described using the first four moments: mean, standard deviation, kurtosis and asymmetry. LPM is for moments what semivariance is for variance, as LPM is based on only one side of the distribution.

While LPM and semivariance focus on returns below a target or the average, respectively, the other downside risk metrics, value at risk (VaR) and conditional value at risk (CVaR) or expected shortfall (ES) focus on extreme negative returns or tail risk. According to Dowd (2002), the VaR is the maximum likely financial loss within a given period and confidence interval provided that there is no certainty of an extreme adverse event. The CVaR of Rockafeller and Uryasev (2000) is the expected loss conditional on the occurrence of an extreme adverse event.

Empirical studies comparing and ranking different risk metrics usually consider the out-of-sample return of portfolios optimized for various risks but with the same return in the sample period. Brazilian studies indicate that LPM-optimized portfolios generally yield the best results, independent of the normality of the distribution of returns; see Andrade (2006) and Araújo and Montini (2015). In light of the literature, *H1*, to be tested is as follows:

H1. Post-modern theory is used more than the modern theory in performance evaluation and in optimization and risk budgeting with absolute risk.

Risk budgeting is a risk management practice in which the portfolio is changed whenever necessary to contain the predicted risk within predetermined limits. Examples of absolute risk are variance, semivariance and VaR.

In addition to the absolute risk metrics described above, relative risk metrics, which measure the variability of the differences between portfolio returns and a benchmark, are essential for the evaluation of investment managers; see Roll (1992). Relative risk metrics are of interest both to passive portfolio managers who aim to replicate the returns of an index or benchmark and to active portfolio managers. Metrics such as tracking error may be used to verify with statistical significance whether a fund with active management is adding value relative to a passive portfolio:

$$TE_p = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}\left(D_n - \overline{D}\right)^2} \cdot \sqrt{T} \qquad (1)$$

where TE_p is the tracking error, N is the number of observations, n is the index of the sum, T is the sample frequency, $D_n = R_{portfolio,n} - R_{benchmark,n}$ and $\overline{D}$ is the average of the return differences. Thus, we have:

H2. The use of relative risk is independent of whether the asset manager has passive indexed funds.

2.2 Covariance matrix estimation

The estimation of portfolio risk typically uses the covariance matrix, which contains the variance of each asset, as well as the covariance for all combinations of two assets, according to Dowd (2002). The covariance matrix is a key part of both portfolio optimization, which aims to obtain the optimal weights of assets, and risk management, which aims to estimate the expected risk of a portfolio or the portfolio's VaR, as in Scherer (2002).

RiskMetrics, proposed by Guldimann *et al.* (1996), uses an exponentially weighted moving-average approach and essentially has only one difference relative to the sample covariance matrix; that is, most recent observations receive greater weight. An exponentially weighted moving-average or EWMA conditional covariance matrix Σt is given by:

$$\Sigma_t = (1 - \lambda)R_{t-1}R'_{t-1} + \lambda \Sigma_{t-1} \tag{2}$$

where R_t is the vector of asset returns at time t, λ is the decay factor, $0 < \lambda < 1$ and the recommended value is $\lambda = 0.94$ for daily data.

Stein (1956) showed that the individually estimated sample mean is not a good estimator of the population mean when it has a multivariate normal distribution. The implication is that in portfolio construction, the sample-based covariance matrix may have estimation errors, although it is not biased. In the same paper, the author suggests using a statistical method to reduce estimation errors, which results in the use of an estimator constructed from the unbiased estimator and a biased and structured estimator, which is subjective and based on previous knowledge and experience.

According to Ledoit and Wolf (2003), the shrinkage method for the covariance matrix is based on the work of Stein (1956) and entails the adjustment of the sample covariance matrix, which typically presents more extreme coefficients, toward a fixed target known as the biased estimator, which ideally has more central values. Frost and Savarino (1986) and Jorion (1986) initially applied shrinkage in the construction of portfolios, but the method did not become practical and feasible until the work of Ledoit and Wolf (2003). We can represent the covariance matrix as:

$$\Sigma_t = \alpha F_t + (1 - \alpha)\hat{\Sigma}_t \tag{3}$$

where F_t is the fixed target or biased estimator, $\alpha \in [0,1]$ and can be interpreted as the weight given to the biased estimator and $\hat{\Sigma}_t$ is the sample covariance matrix.

An alternative way to use the methods suggested above for the covariance matrix involves the use of factors and is based on the theoretical framework such as Ross's (1976) arbitrage pricing theory (APT) and empirical studies such as those of Fama and French (1992). In APT, the excess return of an asset is:

$$R_i - R_f = \sum_{n=1}^{k} \beta_n (f_n - R_f) \tag{4}$$

where there are k systematic risk factors, R_i is the rate of return of asset i, R_f is the risk-free rate, β_n is the sensitivity or load of asset i on risk factor n and f_n is the risk premium of factor n. Assuming that the factors are not correlated with the residual return and that the residual returns are not correlated, the covariance matrix for N assets according to Chan *et al.* (1999) is:

$$\Sigma = B\Omega B^{'} + D \qquad (5)$$

where B is the factor sensitivity matrix, Ω is the covariance matrix of the factors and D is a diagonal matrix containing the residual variances of the return.

Statistical factors are also known as implicit factors, as they are hypothetical variables constructed to explain the movement of a set of time series of returns; see Dowd (2002). Principal component analysis or factor analysis (FA) can be used to identify factors, and both of these quantitative methods can identify independent sources of movement. One of the advantages of using implicit factors in calculating the covariance matrix is to reduce the computational burden since risk factors are independent and a small number of factors are sufficient to achieve high explanatory power; see Alexander (2001) and Amenc and Martellini (2002).

In addition, Engle (1982) and Bollerslev (1986) introduced the univariate generalized autoregressive conditional heteroskedasticity (GARCH) model. Some versions of the multivariate model can dynamically estimate expected and conditional variance and covariance for portfolio assets, and these approaches are recommended when asset volatility is inconsistent over time.

Empirical studies such as those of Santos and Tessari (2012) and Beltrame and Rubesam (2013) point out that the covariance matrix shrinkage method, which combines sample covariance with structured estimators, yields the best results among all the methods. The third test of theory versus practice thus involves testing.

H3. The method most commonly used to determine the covariance matrix is shrinkage.

Studies in Brazil, such as Santos and Tessari (2012), show that minimum variance portfolios obtained through quantitative optimization using the sample covariance matrix, RiskMetrics, explicit factors and GARCH generally have higher returns than an equal-weighted portfolio and a market cap weighted portfolio. Although authors such as DeMiguel *et al.* (2009) obtained different results when studying listed companies in the USA, we consider *H4* as follows:

H4. Investment managers use optimization methods more than the simple rule of establishing a maximum concentration limit per asset.

We do not detail the specific methodology used to optimize portfolios for a risk metric given a covariance matrix. Basically, this entails the optimization of a function subject to constraints. The methodology for portfolio optimization can be found in the studies of Roman and Mitra (2009) and Araújo and Montini (2015).

According to Scherer (2002), managers who are constrained by a risk budget should perform portfolio optimization. The selection of assets and definition of their weights through optimization indicate to managers the optimal portfolio for the pre-established risk limit. Thus, we have:

H5. Investment managers who comply with a risk budget tend to perform portfolio optimization more than those who do not.

2.3 Distribution of returns

According to Roman and Mitra (2009), research on return distributions is as important as the study of risk measures and covariance matrices in maximizing returns or minimizing

risks in a portfolio. The practical implications are better estimates of the parameters for the optimization of financial models, such as the mean and variance optimization of Markowitz or the *capital asset pricing model* (CAPM), and for option pricing models, according to Leal and Ribeiro (2002).

One way to test the suitability of a return distribution is to use the VaR and the Kupiec failure ratio test, which essentially consists of counting the number of losses above the established VaR using out-of-sample testing.

Historical VaR assumes that the distribution of future returns is non-parametric and is based exclusively on historical data or, in other words, that the past provides all the information and the future probability distribution will be equal to the past distribution. The Monte Carlo VaR assumes that future returns will follow a known stochastic process and a non-parametric distribution. To obtain historical or Monte Carlo VaR at the fifth percentile (or a 95 per cent confidence interval), the historical or simulated returns are sorted in ascending order, and the return at the fifth percentile is the estimated VaR.

The simplest parametric VaR assumes that future returns will follow a normal, or Gaussian, parametric distribution that can be estimated with only two parameters: the mean and the expected standard deviation of the returns.

VaR with higher moments considers the existence of kurtosis, asymmetry or both in the distribution and is a more appropriate parametric method when returns do not follow a normal distribution. According to Dowd (2002), one way to consider kurtosis in a distribution is through the Student's t distribution, which has a kurtosis equivalent to 3 $(v - 2)/(v - 4)$, where v can be chosen based on the expected kurtosis of future returns such that $5 < v < 9$.

Extreme value theory (EVT) provides a basis for modeling events with significant economic consequences and very small probabilities of occurring. The generalized extreme value distribution is parametric and may be used to build the worst-case scenario in ten years in the equity market or the probability of the euro-dollar exchange rate appreciating more than 20 per cent in a week.

Brazilian and international empirical studies such as Cassettari (2001), Leal and Ribeiro (2002) and Arraes and Rocha (2006) show that financial asset returns demonstrate asymmetry and kurtosis, and therefore, VaR with a normal distribution is not the most appropriate method. *H6* will test whether the methods suggested by empirical studies in academia are the most frequently used by practitioners but will focus on return distributions:

> *H6.* investment managers use parametric distributions, such as EVT or distributions with higher moments, more than they use a normal distribution to estimate VaR.

2.4 Management of estimation risk

Scherer (2002b) pointed out that the portfolio optimization process suffers from error maximization since assets with higher returns, low risk or low covariance tend to be chosen. Extreme results have a higher probability of estimation errors; in other words, they tend to be unsustainable in the long term. In addition, there is a consensus among scholars about the inability to predict future asset returns and a belief that the optimization process is very sensitive to differences in the expectation of future returns (Michaud, 1989). Some alternatives proposed to address estimation risk are presented below.

The Monte Carlo resampling of Michaud (1989) and Michaud and Michaud (2008) typically uses the draw-with-replacement method to simulate asset returns based on their historical distribution. Usually, the simulation is performed hundreds of times and for all assets to test investment strategies.

Bayesian models and the Black Litterman model – see Jorion (1986) and Black and Litterman (1992) – allow us to add subjective individual convictions to the quantitative financial models. The model will make recommendations after confidence levels for the quantitative model and the subjective individual conviction are determined.

A different strategy to reduce estimation risk involves maintaining a portfolio with the lowest expected risk. The minimum variance portfolio depends only on the covariance estimation and is subject to a more moderate estimation error than other mean-variance portfolios are; see the studies of Chan *et al.* (1999); DeMiguel *et al.* (2009) and Caldeira *et al.* (2013). Given the importance of adopting techniques to manage estimation risk, we thus test:

H7. Estimation risk management methods are more commonly used than the simple rule of imposing a maximum concentration per asset.

2.5 Performance evaluation

The use of risk-adjusted returns to evaluate performance was proposed at nearly the same time as the CAPM model. Treynor (1965); Sharpe (1966) and Jensen (1968) proposed risk-adjusted performance measures based on the theoretical framework of Markowitz's mean-variance model. The desire for a portfolio optimization method to account for market risk and investment analysts' opinions led Treynor and Black (1973) to create a ratio that would later be known as the information ratio. Last on our list of performance measures that are based on the modern portfolio theory is the Modigliani and Modigliani (1997) ratio, which can be interpreted as the return that a fund would have if its risk were equivalent to market risk.

Among the performance metrics that are based on downside risk or post-modern portfolio theory, the Sortino and Van Der Meer (1991) ratio and the return relative to the VaR are worth noting. In the table below, it is important to note that for the Sortino ratio, an investor can replace the risk-free return with a minimum acceptable return; the LPM can replace the semivariance, and the investor can define the degrees of freedom. Dowd (2000) proposed the return relative to VaR and defined it as the return above the risk-free rate divided by the portfolio's VaR (Table I).

Estimating the alpha of a portfolio is not restricted to the method proposed by Jensen (1968), where the alpha is the portfolio return adjusted for the market risk incurred. According to Bailey (1992), the simplest method to estimate alpha is to compare the returns obtained with other similar funds. Fama and French (1992) suggested capturing alpha through multifactor models that consider, for example, size and value factors. Performance attribution and style analysis facilitate the decomposition of excess returns into various components. These components may include, for example, asset class allocation and stock selection or allocation to risk factors (BARRA, 1990; Sharpe, 1992).

H8 refers to the popularity of risk-adjusted returns:

H8. Risk-adjusted returns are used more than unadjusted returns.

Name	Risk metric	Portfolio theory	Formula
Absolute return	No risk adjustment	None	R_p
Excess return on benchmark	No direct risk adjustment	None	$R_p - R_b$
Sharpe ratio	Standard deviation	Modern*	$\dfrac{R_p - R_f}{\sigma_p}$
Treynor ratio	Beta CAPM	Modern	$\dfrac{R_p - R_f}{b_p}$
Jensen's alpha	Beta CAPM	Modern	$R_p - (R_f + b_p(R_m - R_f))$
Information ratio	Standard deviation of residual or *tracking error*	Modern*	$\dfrac{\alpha_p}{\sigma_{\varepsilon p}}$
Modigliani and Modigliani – M^2	Beta CAPM	Modern*	$\left(\dfrac{R_p - R_f}{\sigma_p}\right)\sigma_m - R_f$
Sortino ratio	Semivariance	Post-modern	$\dfrac{R_p - R_f}{\sqrt{SV_p}}$
Return relative to VaR	VaR	Post-modern	$\dfrac{R_p - R_f}{VaR}$

Note: *The empirical tests reported in Section 4 compare the adoption of these performance measures based on the modern theory of portfolios with the adoption of the Sortino index and the return relative to VaR

Table I.
Performance metrics

3. Empirical methodology

To test the hypotheses, we collected data via online questionnaires hosted on the website of the *Centro de Estudos em Finanças* CEF of Fundação Getúlio Vargas' Business Administration School. Invitations and links were sent to key executives, portfolio and risk managers representing 274 asset management companies between August and September 2015. To increase the number of responses, follow-up telephone calls were made to the managers. We obtained telephone numbers and email addresses from the CEF database.

The questionnaire had a total of 3 sections and 17 questions that required the analysis of 65 non-mutually exclusive responses. The questionnaire is based on the work of Amenc *et al.* (2011), who surveyed investment managers in Europe in 2007 on portfolio management and performance evaluation. Although based on a previous survey, a pre-test of the questionnaire was conducted with a group of investment managers with the objective of evaluating the terminology, the clarity of the questions and the average response time, with the main goal of improving the questionnaire.

All questionnaires, according to Graham and Harvey (2001), are subject to potential problems; for example, the responses may not reflect respondents' actions. One concern regarding this work is that some respondents could omit certain information for fear of being copied. Therefore, all the questions were obligatory, except for some identification data: name and email, name and origin of the asset management company and whether the asset management company is affiliated with a bank. Ten of the respondents chose not to identify themselves.

In the present study, all variables of interest are categorical, the majority being binary, with only two categories of responses, such as "uses" and "does not use". The first type of test was the independence test of two variables, which was used to compare categorical

responses between two groups of the same sample or population. We performed independence tests, for example, to compare the rates of utilization of different portfolios and risk management methods. The statistic of the independence test, more specifically of the chi-square test, of Pearson, is given by χ^2 (Table II):

$$e_{ij} = \frac{(Total\ of\ Line\ i)(Total\ of\ Column\ j)}{Size\ of\ the\ Sample\ (N)}$$

$$\chi^2 = \sum_i \sum_j \frac{(f_{ij} - e_{ij})^2}{e_{ij}}$$

where e_{ij} is the expected frequency based on the independence hypothesis for the category in Row i and Column j of the contingency table and f_{ij} is the frequency observed for the category in Row i and Column j of the contingency table. When we analyze two groups and binary responses, the test statistic has approximately a chi-square distribution for large samples with one degree of freedom.

Notably, using a Z-test is preferable when one wishes to analyze differences in the proportions of two different populations. An example is the comparison of proportions verified in the present research carried out in Brazil, with the proportions verified in the research of Amenc *et al.* (2011) in Europe. The Z-test uses two normal distributions rather than independence tests. In cases where the alternative hypothesis is the inequality of proportions and where the chi-square distribution has one degree of freedom, both the Z-test and the chi-square test produce exactly the same *p*-values and the same statistical inferences.

Agresti (1996) suggested the use of Fisher's exact test, which is based on an exact distribution, for statistical inference when the sample is small and the expected value is less than five in any of the categories of the contingency table. To ensure robustness, Fisher's exact test was performed to confirm the chi-square test results, and because we did not find significant differences, we omitted the results of Fisher's exact tests.

The second type of test used multiple regressions to reveal the determinants of the use of quantitative optimization and risk budgeting by investment managers. OLS regressions, known as linear probability models when dependent variables are binary, can be used to reveal causal relationships, according to Angrist and Pischke (2009). We use the probit model for robustness tests. In both regressions, our interest lies mainly in the probability that an observation with particular characteristics will fall into one of two categories:

$$P(y = 1|x_1, x_2, \ldots, x_n) \tag{6}$$

Contingency table	Column variable (groups)		
Line variable (answers)	1	2	Total
1: Uses	X_1	X_2	X
2: Does not use	$n_1 - X_1$	$n_2 - X_2$	$n - X$
Total	n_1	n_2	N

where y is the dependent variable and binary indicator and x_1 is the set of explanatory or independent variables. In the regressions, it is assumed that the probability of response is linearly determined by a set of parameters, and in a probit model, it is also assumed that the probability of response adheres to a standard normal cumulative distribution function. All tests and regressions were conducted using the software Stata version 13.

4. Analysis and discussion of results

4.1 Descriptive statistics and comparison of results with European data

There were 78 respondents; 21 per cent of the asset management companies had a foreign origin, and 29 per cent were affiliated with a bank. Regarding the size of the assets under management, 18 per cent of the respondents play an active role in the management of up to R$250m, 27 per cent between R$250m and R$1bn, 20 per cent between R$1bn and R$5bn and 35 per cent above R$5bn. Regarding the type of managed fund, 62 per cent have active equity funds, 21 per cent indexed equity funds, 26 per cent short-term government bond funds or *fundos DI*, 47 per cent fixed income funds, 73 per cent hedge funds or *fundos multimercados* and 15 per cent other funds.

In Table III, we present a comparison of the results obtained with those of Amenc *et al.* (2011), as well as the *p*-value of the equality of proportions test, which will, in this case, be identical to the *p*-value of the chi-square test, as explained in Section 3 of the paper.

4.2 Adoption of modern and post-modern theory

Three independence tests were performed to compare investment managers' adoption of the modern theory with that of post-modern theory. No statistically significant differences in risk management were found, specifically in the use of risk budgeting; 51.3 per cent are based on the modern theory and 59.0 per cent on the post-modern theory. Statistically significant differences in favor of the modern portfolio theory compared with post-modern theory were found:

- in the construction of portfolios (21.8 per cent versus 6.8 per cent respectively) and
- in performance evaluation (80.8 per cent versus 35.9 per cent, respectively).

Despite the results of academic studies such as Araújo and Montini (2015), the results reported in Table IV lead not only to the rejection of *H1*, but also to the conclusion that the modern theory remains the dominant paradigm among practitioners.

We tested *H1* in several subsamples: large asset management companies, with over R$1bn in assets under management, and small companies; companies with hedge funds and without; companies affiliated with banks and those that are independent; and companies with (a) one or two and three or (b) more types of funds. The results reported in Table V reject *H1* for all subgroups in the sample as they are similar to the ones obtained for the full sample.

4.3 Determinants of relative risk budgeting and portfolio optimization

The importance of relative risk and return for Brazilian investment managers is evident: 75.6 per cent target a return above the benchmark, 64.1 per cent and 48.7 per cent perform, respectively, budgeting and portfolio optimization with relative risk. For comparison purposes, 65.4 per cent aim for the absolute return, and 55.1 per cent and 25.6 per cent perform budgeting and portfolio optimization with absolute risk. We conduct OLS and probit regressions to better understand the determinants of the use of

Observations	Brazil 78 (%)	Europe 229 (%)	Δ (%)	p-value	Sig
In portfolio optimization, are objectives set for absolute risk?					
Yes, variance, volatility or standard deviation	21.8	45.9	−24.1	0.000	***
Yes, VaR or CVaR	2.6	50.7	−48.1	0.000	***
Yes, semivariance or LPM	5.1	23.1	−18.0	0.000	***
In portfolio optimization, are objectives set for relative risk?					
No	51.3	33.6	17.7	0.007	***
Yes, tracking error	34.6	49.8	−15.2	0.025	**
Yes, tracking error VaR or BVaR	19.2	18.8	0.5	0.930	
Yes, semivariance or LPM	11.5	12.7	−1.1	0.794	
Yes, other	0.0	2.6	−2.6	0.149	
How is the covariance matrix estimated?					
Sample covariance or RiskMetrics	53.8	59.8	−6.0	0.355	
Explicit factors, such as CAPM	19.2	29.3	−10.0	0.084	
Factor analysis or principal component analysis	3.8	12.7	−8.8	0.028	**
Shrinkage methods	1.3	3.9	−2.6	0.255	
GARCH and others	17.9	7.9	10.1	0.012	**
How is VaR estimated?					
Normal distribution	62.8	41.0	21.8	0.001	***
Higher moments	7.7	16.6	−8.9	0.053	*
Extreme value theory	5.1	8.3	−3.2	0.359	
CVaR or expected shortfall	29.5	22.3	7.2	0.198	
Other	15.4	12.7	2.7	0.542	
What methods are used to address estimation risk?					
Maximum concentration limit per asset	64.1	67.7	−3.6	0.562	
Global minimum variance portfolio	5.1	17.0	−11.9	0.009	***
Bayesian methods	7.7	15.3	−7.6	0.089	
Resampling	9.0	13.5	−4.6	0.291	
Other	9.0	6.1	2.9	0.387	
How is performance evaluated?					
Sharpe ratio	78.2	77.3	0.9	0.868	
Treynor ratio	7.7	10.9	−3.2	0.414	
Sortino ratio	14.1	27.5	−13.4	0.017	**
Absolute return	65.4	41.5	23.9	0.000	***
Modigliani and Modigliani	5.1	3.1	2.1	0.395	
Jensen's alpha	15.4	34.1	−18.7	0.002	***
Information ratio	19.2	65.1	−45.8	0.000	***
Excess return relative to benchmark	75.6	31.4	44.2	0.000	***
How is alpha measured?					
Multifactor models	9.0	21.4	−12.4	0.014	**
Single-factor models (such as CAPM)	17.9	26.6	−8.7	0.123	
Performance attribution or style analysis	69.2	35.4	33.9	0.000	***
Peer group analysis	66.7	56.8	9.9	0.124	
Other	5.1	2.2	2.9	0.183	

Notes: The p-values provided are from Pearson's chi-square test with one degree of freedom. The null hypothesis is the equality of proportions, and the alternative hypothesis is inequality. We reject the null hypothesis at $p < 0.01$ (***); $p < 0.05$ (**) and $p < 0.1$ (*)

Table III.
Comparison of
selected responses
for Brazil vs Europe

ID	Hypothesis	Test	Result		
H1	Post-modern theory is used more than the modern theory in performance evaluation and in optimization and risk budgeting with absolute risk	Chi-square statistic	Hypothesis rejected based on tests in full sample and subsamples		
	Tests performed	Hypotdeses	*p*-value	Sig	Result
T1a	Comparison of the proportions of respondents using modern ($\theta 1$) and post-modern ($\theta 2$) theory for absolute risk budgeting	h0: $\theta_1 = \theta_2$ h1: $\theta_1 > \theta_2$	0.334		*H0* Not Rejected
T1b	Comparison of the proportions of respondents using modern ($\theta 1$) and post-modern ($\theta 2$) theory for portfolio optimization with absolute risk	h0: $\theta_1 = \theta_2$ h1: $\theta_1 > \theta_2$	0.006	***	*H0* Rejected
T1c	Comparison of the proportions of respondents using modern ($\theta 1$) and post-Modern ($\theta 2$) Theory for performance evaluation	h0: $\theta_1 = \theta_2$ h1: $\theta_1 > \theta_2$	0.000	***	*H0* Rejected

Table IV.
Description of *H1*
variables, tests and
results

		T1a		T1b		T1c	
Subsample	Obs	*p*-value	Sig	*p*-value	Sig	*p*-value	Sig
Large	43	0.822		0.024	**	0.000	***
Small	35	1.000		0.101	*	0.001	***
With hedge funds	57	0.338		0.003	***	0.000	***
Without	21	0.753		1.000		0.002	***
Banks	20	0.490		0.633		0.010	**
Independent	46	0.835		0.036	**	0.000	***
1 or 2 funds	32	0.035	**	0.03	**	0.002	***
3 or more funds	46	0.676		0.079	*	0.000	***
Total	78	0.334		0.006	***	0.000	***

Table V.
Results of *H1* chi-
square tests on
subsamples

Notes: The *p*-values provided are from chi-square tests of independence. The null hypothesis is equal use of modern and post-modern theory. T1a, T1b and T1c, respectively, address absolute risk budgeting, absolute risk optimization and performance evaluation. We reject the null hypothesis at $p < 0.01$ (***); $p < 0.05$ (**); $p < 0.1$ (*)

relative risk and to test *H2*. The same approach was used to analyze the determinants of portfolio optimization and to test *H5*; see Table VI.

In Models (1) and (2) in Table VII, the dependent variable is binary with a value of 1 (one) when the manager has a budget for relative risk and 0 (zero) when there is no such budget. In the other models, the dependent variables are also binary and take a value of one when the method is adopted and zero otherwise. The dependent variable is portfolio optimization with relative risk in Models (3) and (4) and is optimization with absolute risk in Models (5) and (6). Models (1)-(4) will be used to test *H2* and Models (3)-(6) to test *H5*.

The independent variables are as follows: indexed equity funds, which is an indicator variable for whether or not the manager has passive indexed equity funds; active equity funds; large, if the assets under management are above R$1bn; foreign,

ID	Hypotheses	Tests	Results			
H2	The usage of relative risk is independent of whether the manager has passive indexed funds	Coefficient of the variable of interest in the regression	Rejected			
H5	Managers who comply with a risk budget tend to optimize their portfolios	Coefficient of the variable of interest in the regression	Partially rejected			

Regression equations and tests performed

Model	*Regression equations*	Hypotheses	*p*-value	Sig	*Results*
(1)	$\pi(x) = \alpha + \beta_1 x + \gamma\Gamma$: where $\pi(x)$ is the probability of setting a budget based on relative risk and x is the presence of indexed funds. See legend below	h0: $\beta_1 = 0$ h1: $\beta_1 \neq 0$	0.096	*	*H0* Rejected
(2)	*probit*$[\pi(x)] = \alpha + \beta_1 x + \gamma\Gamma$: see legend in (1)	h0: $\beta_1 = 0$ h1: $\beta_1 \neq 0$	0.099	*	*H0* Rejected
(3)	$\pi(x) = \alpha + \beta_1 x + \beta_2 w + \gamma\Gamma$: where $\pi(x)$ is the probability of performing relative risk optimization, x is the presence of indexed equity funds and w is the use of relative risk budgeting. See legend below	h0: $\beta_1 = 0$ h1: $\beta_1 \neq 0$ h0: $\beta_2 = 0$ h1: $\beta_2 \neq 0$	0.058 0.000	* ***	*H0* Rejected *H0* Rejected
(4)	*probit*$[\pi(x)] = \alpha + \beta_1 x + \beta_2 w + \gamma\Gamma$: see legend in (3)	h0: $\beta_1 = 0$ h1: $\beta_1 \neq 0$ h0: $\beta_2 = 0$ h1: $\beta_2 \neq 0$	0.046 0.000	** ***	*H0* Rejected *H0* Rejected
(5)	$\pi(x) = \alpha + \beta_1 v + \gamma\Gamma$: where $\pi(x)$ is the probability of performing absolute risk optimization and v is the presence of an absolute risk budget. See legend below	h0: $\beta_1 = 0$ h1: $\beta_1 \neq 0$	0.800		*H0* Not rejected
(6)	*probit*$[\pi(x)] = \alpha + \beta_1 v + \gamma\Gamma$: see legend in (5)	h0: $\beta_1 = 0$ h1: $\beta_1 > 0$	0.571		*H0* Not rejected

Notes: We reject the null hypothesis of $\beta = 0$ at $p < 0.01$ (***); $p < 0.05$ (**); $p < 0.1$ (*). The probit $[\pi(x)]$ is a function or link that transforms $\pi(x)$ into a Z statistic, α is the constant, β_1 and β_2 are the coefficients of interest, Γ is the vector of control variables and γ is the vector of coefficients of the control variables

Table VI.
Description of *H2* and *H5* variables, tests and results

which describes the origin of the asset management company; bank, if the asset management company is affiliated with a private or public bank; hedge funds, which includes any type of hedge fund; and budgeting with relative and absolute risk indicates whether the manager sets a risk budget or not.

With regard to the regression results, Models (1) and (2) indicate that the only variable that explains relative risk budgeting with statistical significance is whether investment managers have passive or indexed equity funds. The presence of an index equity fund increases the probability of using a relative risk budget by 29.6 per cent according to Model (1) OLS and by 31.2 per cent according to Model (2) probit. All marginal probabilities with probit are estimated considering the other variables at their mean.

Models (3) and (4) of Table VII have significant explanatory power. The R^2 and pseudo R^2 values for the probit model are 0.50 and 0.55, respectively. The models show

Independent variables	Budgeting with relative risk		Dependent variables Optimization with relative risk		Optimization with absolute risk	
	(1) OLS	(2) PROBIT	(3) OLS	(4) PROBIT	(5) OLS	(6) PROBIT
Indexed equity funds	0.293* (0.173)	1.064* (0.644)	0.324* (0.168)	1.772** (0.889)	0.0231 (0.126)	0.0416 (0.392)
Active equity funds	−0.0936 (0.138)	−0.242 (0.369)	−0.0300 (0.0902)	−0.328 (0.455)	−0.218* (0.126)	−0.768* (0.462)
Large	−0.0646 (0.154)	−0.157 (0.399)	0.0504 (0.104)	0.242 (0.448)	0.125 (0.153)	0.431 (0.485)
Foreign	0.131 (0.132)	0.451 (0.477)	−0.111 (0.141)	−0.321 (0.622)	0.0198 (0.128)	0.120 (0.465)
Bank	0.130 (0.138)	0.367 (0.440)	−0.0398 (0.0940)	−0.531 (0.622)	0.213** (0.104)	0.814* (0.455)
Hedge funds	0.104 (0.131)	0.267 (0.361)	−0.0915 (0.0825)	−0.667 (0.499)		
Budgeting with relative risk			0.692*** (0.0914)	2.798*** (0.511)		
Budgeting with absolute risk					−0.0269 (0.106)	−0.197 (0.348)
Constant	0.530*** (0.142)	0.0618 (0.384)	0.0979 (0.0779)	−1.516*** (0.492)	0.164 (0.141)	−1.019* (0.526)
Observations	68	68	68	68	68	68
R^2 or probit's pseudo R^2	0.128	0.112	0.547	0.499	0.086	0.083

Notes: Robust standard errors are in parentheses. We reject the regression coefficient $\beta = 0$ at $p < 0.01$ (***); $p < 0.05$ (**); $p < 0.1$ (*)

that both the presence of indexed equity funds and the use of relative risk budgets explain optimization with relative risk. The presence of indexed funds increases the probability of relative risk optimization from 32.4 per cent in the OLS model (3) to 59.3 per cent in the probit model (4). The use of relative risk budgeting increases the probability of relative risk optimization from 69.2 per cent in the OLS Model (3) to 77.0 per cent in the probit model (4).

Two variables explain portfolio optimization with absolute risk with statistical significance, according to Models (5) and (6) in Table VII. Neither of them is absolute risk budgeting. Respondents with hedge funds are 21.3-20.2 per cent more likely than respondents without hedge funds to perform this procedure, according to the (5) OLS and (6) probit models, respectively. Respondents from large companies are between 21.8 per cent and 22.4 per cent less likely than those from small companies to perform absolute risk optimization, according to Models (5) and (6), respectively.

Before we conclude, it is important to make three brief technical notes and address some endogeneity concerns. First, the probit model assumes that the errors in the probability of response follow a normal distribution. The LPM model is more appropriate in our case because it is more robust to specification errors, and because the independent variables are all binary, all estimated probabilities will be within the range of [0,1]; see Angrist and Prischke (2009). Second, the addition of relative risk budgeting and indexed equity funds as independent variables in the same regression generates multicollinearity because both are significantly correlated. This may increase the standard errors, making it harder to detect statistical significance in the regressors, but it will not bias the regression coefficients. Finally, the constants of regressions (1) to (6) theoretically indicate the probability of an asset management company performing a risk budget or optimization if the company is small and local; has no index, active equity or hedge funds; and is not affiliated with a bank. In practice, however, there is arguably considerable noise in the determination of the constants, and therefore, we will refrain from interpreting the constants.

It could be argued that the use of budgeting and optimization is what causes an asset management company to be large, which would create endogeneity due to reverse causality. The argument that investors seek risk-efficient portfolios is plausible and supported by studies in the USA. Ippolito (1992) and Sirri and Tufano (1998) found a negative impact of risk on fundraising. However, only a marginal impact was found using US data, and Muniz (2015) found no impact at all in Brazil.

In Models (3)-(6) of Table VII, it is unlikely that optimization causes the risk budget and that models suffer from reverse causality. It seems more plausible that investment managers who need to adhere to risk limits imposed by customers rely on optimization to help them determine the portfolio with the highest expected return for a certain amount of risk.

Finally, since the presence of an indexed or passive equity fund increases the probability of budgeting or optimizing with relative risk, the 16 respondents who had equity index funds are more likely to use relative risk than the remaining respondents, and thus, *H2* can be rejected.

As relative risk budgeting explains optimization with relative risk but absolute risk budgeting does not explain optimization with absolute risk, we can partially reject *H5*. Risk budgeting was found to affect only optimization with relative risk, not optimization with absolute risk.

Although we do not have all the necessary elements, we will speculate on a possible explanation for the partial rejection of *H5*. It may be the case that there are simple methods, such as limiting the maximum concentration per individual asset, that work sufficiently well

for the management of absolute risk, but the same does not occur for the management of relative risk. An alternative, albeit longer, explanation is that it is well known that performance fees to asset management companies are generally tied to the excess return relative to the benchmark. Investment managers who do not perform optimization with relative risk may achieve lower excess returns, bring less revenue to the asset management company or fall outside of the pre-established risk limits. This would culminate in their dismissal, and investment managers who remain are those who optimize with relative risk. However, this may not be the case for portfolio managers who do not perform optimization with absolute returns, as performance fees are not usually linked to absolute returns.

4.4 Quantitative optimization and return distribution

Starting with *H3*, while only 1.3 per cent uses the shrinkage matrix, 34.6 per cent use RiskMetrics, the most popular method to produce a covariance matrix. Independence tests reject *H3*. Despite the excellent results in empirical tests reported by Ledoit and Wolf (2003) and Santos and Tessari (2012), even in Europe, the method is used by only 3.9 per cent of managers (Table III). Attributing this low adoption to a costly learning curve does not seem reasonable since models such as GARCH are possibly more complex and are used by 17.9 per cent of managers.

Imposing asset concentration limits is a simple and intuitive method but is relatively arbitrary and without scientific backing, according to Amenc *et al.* (2011). *H4* tests whether this method is more popular than quantitative optimization. Of the respondents, 55.1 per cent performed optimization by absolute, relative risk or both, whereas 64.1 per cent set restrictions on asset concentration. The difference between the two proportions is not statistically significant, and we thus reject *H4*.

Only 5.1 per cent of managers use EVT, 7.7 per cent use distributions with higher moments and 9.0 per cent use at least one of the two methods to estimate VaR. Because 62.8 per cent of the sample, a much higher percentage, uses the normal distribution, we can reject *H6*. See Table VIII. According to Cassettari (2001) and Leal and Ribeiro (2002), the preference for a normal parametric distribution is probably associated with simplicity and ease of use, but the asymmetry of the return distribution in the Brazilian market implies that simplification may lead to underestimation of the tail risk and may cause unpleasant surprises. Surprisingly, EVT and higher moments are more common in developed countries, where the return distribution resembles more a normal curve, than in Brazil (8.3 per cent and 16.6 per cent of respondents, respectively), according to Table III.

Investment managers in Brazil are much less likely to adopt the selected methods to manage estimation risk than are their counterparts in Europe. The respective values are 5.1 per cent versus 17.0 per cent that adopt minimum variance portfolios, 7.7 per cent versus 15.3 per cent that adopt Bayesian methods and 9.0 per cent versus 13.5 per cent that adopt resampling. See Table III. Only 24.4 per cent use some method to manage estimation errors, thus leading us to reject *H7*. The results are robust for the subsample of 43 respondents who perform optimization with relative or absolute risk; see T7b in Table VIII.

4.5 Risk-adjusted versus unadjusted return

Previous tests suggest that there is a significant gap between best practices and current practices in the Brazilian asset management industry. One could argue that the comparison is of little use, as the ideas conveyed in academic publications will always be years ahead of the market. If best practices take years to be fully understood, disseminated and implemented and new ideas are constantly being created, then a gap should naturally be expected (Table IX).

ID	Hypotheses	Tests	Results			
H3	The method most commonly used to determine the covariance matrix is shrinkage	Chi-square statistic	Hypothesis rejected			
H4	Investment managers use optimization methods more than the simple rule of establishing a maximum concentration limit per asset	Chi-square	Hypothesis not rejected			
H6	Investment managers use parametric distributions, such as extreme value theory or distributions with higher moments, more than the normal distribution to estimate VaR	Chi-square	Hypothesis rejected			
H7	Estimation risk management methods are more commonly used than the simple rule of imposing a maximum concentration per asset	Chi-square	Hypothesis rejected			
	Tests performed	Hypotheses	p-value	Sig	Results	
T3	Comparison of the proportions of respondents using shrinkage ($\theta 1$) and those using RiskMetrics ($\theta 2$) in determining the covariance matrix	H0: $\theta_1 = \theta_2$ h1: $\theta_1 < \theta_2$	0.000	***	$H0$ Rejected	
T4	Comparison of the proportions of respondents using quantitative optimization ($\theta 1$) and those using maximum concentration per asset ($\theta 2$) in the construction of portfolios	H0: $\theta_1 = \theta_2$ h1: $\theta_1 > \theta_2$	0.253		$H0$ Not rejected	
T6	Comparison of the proportions of respondents using extreme value theory or a distribution with upper moments ($\theta 1$) and those using the normal distribution ($\theta 2$)	H0: $\theta_1 = \theta_2$ h1: $\theta_1 < \theta_2$	0.000	***	$H0$ Rejected	
T7	Comparison of proportions of respondents using advanced estimation risk management methods ($\theta 1$) and those using maximum concentration per asset ($\theta 2$)	h0: $\theta_1 = \theta_2$ h1: $\theta_1 < \theta_2$	0.000	***	$H0$ Rejected	
T7b	Same variables as T7. Test performed in the subsample of 43 respondents who perform portfolio optimization	h0: $\theta_1 = \theta_2$ h1: $\theta_1 < \theta_2$	0.000	***	$H0$ Rejected	

Note: We reject the null hypothesis at $p < 0.01$ (***)

Table VIII.
Description of H3, H4, H6 and H7 variables, tests and results

ID	Hypotheses	Tests	Results			
H8	Risk-adjusted returns are used more than unadjusted returns	Chi-square statistic	Hypothesis rejected			
	Tests performed	Hypotheses	p-value	Sig	Results	
T8	Comparison of proportions of respondents using risk-adjusted ($\theta 1$) and those using unadjusted ($\theta 2$) returns	h0: $\theta_1 = \theta_2$ h1: $\theta_1 < \theta_2$	0.151		$H0$ not rejected	

Table IX.
Description of H8 variables, tests and results

The analysis of risk-adjusted returns was introduced in Treynor (1965); Sharpe (1966) and Jensen (1968) and cannot be considered new. Furthermore, it cannot be considered very complex, since in Europe, according to Amenc *et al.* (2011), 77 per cent of managers use the Sharpe index and 65 per cent use the information ratio. They are the most commonly used metrics to evaluate performance in Europe, according to Table III.

Although it is relatively old and not very complicated concept, the risk-adjusted return is not as popular as the unadjusted return in Brazil, according to the survey, and we can thus reject *H8*. Of the Brazilian managers, 91.0 per cent use absolute return or excess return relative to the benchmark; these are measures of return not directly adjusted for risk. In comparison, 83.3 per cent use at least one risk-adjusted return measure such as the Sharpe or information ratio.

5. Final remarks

This paper aims to identify a possible mismatch between the theory found in academic research and the practices of investment managers in Brazil. For this purpose, a bibliographical and field survey was carried out with 78 respondents to a questionnaire posted online, out of a total of 274 asset management companies. This study may be considered a pioneering work in portfolio construction, risk management and performance evaluation in Brazil.

The results of the tests performed indicate that practice departs from theory in the country: of the eight hypotheses tested, we rejected seven hypotheses and partially rejected one hypothesis. One possible explanation is that few Brazilian academic studies consider transaction costs such as brokerage fees, bid-ask spreads and liquidity when studying the benefits of quantitative portfolio optimization. Santos and Tessari (2012) and Caldeira *et al*. (2013), for example, considered three types of rebalancing, daily, weekly and monthly, which would generate very high turnover and cost.

Compared with investment managers in Brazil, those in Europe seem to be closer to the best practices propagated by academia. During the data collection process, some of the respondents mentioned that the low liquidity and quantity of financial assets in Brazil did not justify the use of quantitative methods for portfolio construction. Notably, the European managers researched by Amenc *et al*. (2011) represented companies with greater assets under management overall.

This work aims not only to suggest improvements to practitioners but also to help researchers better understand the reality in Brazil. As an example, although Santos and Tessari (2012) noted that the sample covariance matrix is seldom used by practitioners, only RiskMetrics is more popular than the sample covariance matrix.

Perhaps the most important contribution in this sense is that managers attach great importance to relative risk and return, but few empirical studies focusing on relative risk were found. In future work, we suggest that researchers compare optimized portfolios for tracking error, benchmark VaR and other optimization methods with relative risk. We also suggest studies that help determine how international diversification or the financial education of clients of asset management companies relates to the adoption of sophisticated methods for portfolio construction, risk management, and performance evaluation. Because the present study is limited in that it reflects the state of the industry in only a short time interval, longitudinal studies are suggested because they do not suffer from this restriction.

This paper also aimed to investigate the determinants of the risk budget and the optimization of portfolios. The only variable that significantly explained risk budgeting was the presence of passive or indexed equity funds. For portfolio optimization, the key explanatory variables differed depending on the type of risk: absolute or relative risk. While the presence of passive equity funds and risk budgeting increases the likelihood of relative risk optimization, the presence of hedge funds and assets under management of less than R$1bn increases the probability of absolute risk optimization.

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