

In Memory of Vladimir Shelkovich (1949–2013)

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Vladimir Shelkovich was born in spring 1949 in Rostov-on-Don, a Southern Russian city. After graduation from the Rostov University, he had been working for some time in Rostov and then, in 1980, moved to Leningrad (now Saint Petersburg) and began to work at the St. Petersburg State University of Architecture and Civil Engineering, where he had been working till the last days of his life.

I met Vladimir in the second half of 1980s. At that time, I was already a rather well-known scientist in Russia and a coauthor of several books, and hence, at the very beginning, our relations resembled the relations between a teacher and a pupil, but even at that initial stage of our acquaintance, the influence of Vladimir's ideas was significant. At that time, he was studying the algebraic constructions in the theory of generalized functions on the basis of harmonic approximations. It was clear from the very beginning that this work would not find wide application, but the idea of constructive approach to studying the algebraic structures related to generalized functions was itself very close to the field of my research.

In a sense, our common results were based on these ideas but the harmonic approximations were replaced by smooth approximations (arbitrary or special, which were determined by some equations).

From the very beginning of our acquaintance, Vladimir was my closest friend, and his outstanding talents very soon allowed him to become my full-right coauthor.

As a result of our common work (Georgii Omel'yanov joined us later), we formulated a new approach for constructing the limit problems describing the propagation and (which is most important!) interaction of singularities of solutions to nonlinear partial differential equations. We called this approach the weak asymptotics method. There are rather many methods for constructing approximate analytic (asymptotic) solutions of PDE. All of them are based on the construction of functions depending on a small parameter and satisfying the problem conditions (the equation, initial data,

etc.) not exactly but with an arbitrary, usually polynomial, accuracy in a small parameter. The accuracy (smallness) of the discrepancy is understood in the sense of the maximum of the absolute value. In the weak asymptotics method, we proceed in the same way but the discrepancy smallness is understood in a weak sense.

There is an analogy between the weak asymptotics method, the Maslov–Whitham method for constructing asymptotic solutions with localized fast variation, and the method for constructing solutions of nonlinear equations developed by J.-F. Colombeau.

In the framework of the weak asymptotics method, the delta-shock wave solutions were defined for different systems of conservation laws and the superposition laws were obtained for such solutions. It should be noted that construction of the solutions describing the interaction of nonlinear waves with localized fast variation is a characteristic feature of the weak asymptotics method. Such constructions are impossible in the framework of the Maslov–Whitham method, and in the framework of the J.-F. Colombeau theory, such a construction is technically possible but has not been performed.

In the last years of his life, we did not write joint papers but were close friends. Vladimir was one of two or three peoples who knew the details of my life and I also knew almost everything about him. His untimely demise was absolutely unexpected for me, because I was sure that Vladimir was much younger. On the whole, it was very easy to deal with him, although our temperaments were quite different.

Thinking about our meetings and conversations, I recall his decency, sympathy, extreme sensitiveness to justice, and humor. He could not ignore a sick animal, was always attentive and caring to people. Vladimir’s attitude to his wife Zhenya was very touching, and they brought up their beautiful son Mikhail. Vladimir was a many-sided personality, and his deep knowledge in many areas, even in Hinduism and Buddhism, was fantastic, he also knew Sanskrit (and had been teaching it for a short time !). Vladimir wrote and published a philological paper about Dante’s poem “The Divine Comedy”, and was writing a paper about J. Joyce’s roman “Ulysses”.

His bright life came abruptly to an end, but the blessed memory of Vladimir Shelkovich will forever remain in the hearts of his students, friends, and colleagues.

Mathematical publications of Vladimir M.Shelkovich :

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As a result of our common work (Georgii Omel'yanov joined us later), we formulated a new approach for constructing the limit problems describing the propagation and (which is most important!) interaction of singularities of solutions to nonlinear partial differential equations. We called this approach the weak asymptotics method. There are rather many methods for constructing approximate analytic (asymptotic) solutions of PDE. All of them are based on the construction of functions depending on a small parameter and satisfying the problem conditions (the equation, initial data, etc.) not exactly but with an arbitrary, usually polynomial, accuracy in a small parameter. The accuracy (smallness) of the discrepancy is understood in the sense of the maximum of the absolute value. In the weak asymptotics method, we proceed in the same way but the discrepancy smallness is understood in a weak sense.

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In the last years of his life, we did not write joint papers but were close friends. Note that, in these last years, Vladimir’s scientific interests broadened to the field of p -adic analysis, and he obtained many interesting results there.

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Mathematical publications of Vladimir M. Shelkovich

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